

Name: _____

Midterm #1 – IO (October 27th, 2026)

Question #1

Suppose that there are only two firms selling Jeans, called *Levis (L)* and *True Religion (TR)*. The production is costless the *inverse demand* structure for the two products is:

$$p_L = 10 - 2q_L - 0.5q_{TR} \text{ and } p_{TR} = 10 - 0.5q_L - 2q_{TR} \text{ where } b > 0 \text{ and } b^2 > c^2$$

And the *direct demand* functions are,

$$q_L = 4 - 0.53p_L + 0.13p_{TR} \text{ and } q_{TR} = 4 + 0.13p_L - 0.53p_{TR}$$

- a. Solve the Cournot game with differentiated product. Make sure you solve for the output level of each firm, market price and profits. [Hint: use the **inverse demand** function]

Levis solves the following maximization problem:

$$\text{Max}_{q_L} \pi_L = (10 - 2q_L - 0.5q_{TR}) \times q_L$$

Hence the first order condition with respect to q_L is:

$$\frac{\partial \pi_L}{\partial q_L} = 10 - 4q_L - 0.5q_{TR} = 0$$

Solving for q_L , we obtain Levi's best response function:

$$R_L(q_{TR}) = q_L = \frac{10 - 0.5q_{TR}}{4} \quad (A)$$

By symmetry, True Religion's best response function is:

$$R_{TR}(q_L) = q_{TR} = \frac{10 - 0.5q_L}{4} \quad (B)$$

Substituting (B) into (A),

$$q_L = \frac{10}{4} - \frac{0.5}{4} \times \frac{10 - 0.5q_L}{4} \quad / \times 16$$

$$16q_L = 40 - 5 + 0.25q_L \quad / -0.25q_L$$

$$15.75q_L = 35$$

$$q_L = 2.22$$

And by symmetry, $q_{TR} = 2.22$

$$p_L = p_{TR} = 10 - 2 \times 2.22 - 0.5 \times 2.22 = 4.45$$

$$\pi_L = \pi_{TR} = 4.45 \times 2.22 = 9.879$$

- b. Solve the Bertrand game with differentiated products. Make sure you solve for the market price. [Hint: use the **direct demand** function]

Levi's maximizes the following profit function

$$\text{Max}_{p_L} \pi_L = (4 - 0.53p_L + 0.13p_{TR}) \times p_L$$

Hence the first order condition with respect to p_L is:

$$\frac{\partial \pi_L}{\partial p_L} = 4 - 1.06p_L + 0.13p_{TR} = 0$$

Solving for p_L , we obtain Levi's best response function:

$$p_L = \frac{4 + 0.13p_{TR}}{1.06} \quad (I)$$

And by symmetry, True Religion's best response function is:

$$p_{TR} = \frac{4 + 0.13p_L}{1.06} \quad (II)$$

Substituting (II) into (I),

$$p_L = \frac{4}{1.06} + \frac{0.13}{1.06} \times \frac{4 + 0.13p_L}{1.06} \quad / \times 1.1236$$

$$1.1236p_L = 4.24 + 0.52 + 0.0169p_L \quad / -0.0169p_L$$

$$1.1067p_L = 4.76$$

$$p_L = 4.30$$

By symmetry, $p_{TR} = 4.30$

- c. Which market structure, a Cournot or a Bertrand, would yield a higher market price? How would changing the degree of product differentiation affect the relative difference between the two market-structure outcomes?

$P^C = 4.45 > 4.30 = P^B$, hence Cournot yields a higher market price. The more differentiated the products are, the smaller the difference between the Cournot and Bertrand prices.

Question #2

Consider the following 2x2 game

	Player 2	
	Left	Right
Player 1	Left	(14,10)
	Right	(4,0)

- By inspection, what are the pure strategy Nash equilibria?
- Find the additional mixed strategy equilibrium by using the fact that if a player is willing to mix between two or more strategies, she will be indifferent between them in equilibrium.
- Draw the best-response correspondences. Where do they intersect?

Solution

- $NE = \{(L,L), (R,R)\}$
- Assume p is the probability that player 1 selects L and q is the probability that player 2 selects L.
Hence:

$$EU_1(L) = 14q + 0(1-q) = 14q$$

$$EU_1(R) = 4q + 8(1-q) = -4q + 8$$

Hence player 1 chooses L if $EU_1(L) > EU_1(R)$. That is, if $14q > -4q + 8$ or $q > 4/9$.

In addition,

$$EU_2(L) = 10p + 0(1-p) = 10p$$

$$EU_2(R) = 3p + 8(1-p) = -5p + 8$$

Hence player 2 chooses L if $EU_2(L) > EU_2(R)$. That is, if $10p > -5p + 8$ or $p > 8/15$.

Therefore players' best response functions are :

$$R^1(q) = \begin{cases} R (p=0) & \text{if } q < 4/9 \\ \text{Randomizes between L \& R} & \text{if } q = 4/9 \\ L (p=1) & \text{if } q > 4/9 \end{cases}$$

$$R^2(p) = \begin{cases} R (q=0) & \text{if } p < 8/15 \\ \text{Randomizes between L \& R} & \text{if } p = 8/15 \\ L (q=1) & \text{if } p > 8/15 \end{cases}$$

$$L (p=1) \text{ if } q > 4/9$$

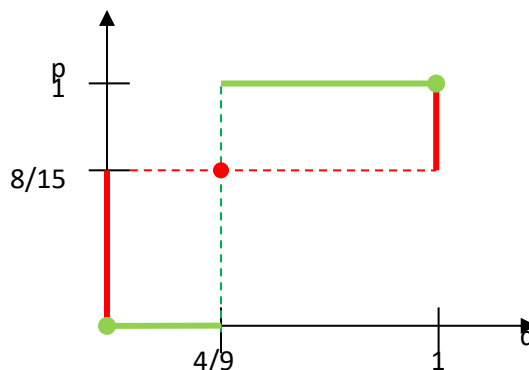
$$R^2(p) = \begin{cases} R (q=0) & \text{if } p < 8/15 \\ \text{Randomizes between L \& R} & \text{if } p = 8/15 \\ L (q=1) & \text{if } p > 8/15 \end{cases}$$

$$\text{Randomizes between L \& R (p} \in (0,1) \text{ if } p = 8/15$$

$$L (q=1) \text{ if } p > 8/15$$

The MSNE is player 1 randomizes between L and R if $q = 4/9$ and player 2 randomizes between L and R if $p = 8/15$

c)



Question #3

An industry consists of two firms with identical costs $C(q)=10q+(q^2)/2$. The firms can either "collude" or "compete." If both collude, they each produce half the monopoly output. If one firm colludes and the other competes, the latter produces the output q that maximizes its profits given that the other firm produces the monopoly output. If both compete, they play Cournot. Calculate these outputs and the resulting profits if market demand is $Q=110-p$. Represent your results as a normal-form game. What is the Nash equilibrium if the game is only played once?

A monopoly with the same costs as the two firms combined would have costs

$$C(Q) = 2 \left[\frac{Q}{2} + \frac{\left(\frac{Q}{2}\right)^2}{2} \right] = 10Q + \frac{Q^2}{4}$$

Its profits would be $\pi = (110 - Q)Q - 10Q - \frac{Q^2}{4}$

Setting the derivative of these profits with respect to Q equal to zero, we find that $Q_m = 40$. Hence, if the firms collude, each produces $q_m = 20$ and earns profits $\pi_m = \Pi_m/2 = \$1,000$.

If only one firm competes, its profits are

$$\tilde{\pi} = (110 - q - \tilde{q})\tilde{q} - 10\tilde{q} - \frac{\tilde{q}^2}{2} = (90 - \tilde{q})\tilde{q} - 10\tilde{q} - \frac{\tilde{q}^2}{2}$$

Setting the derivative of these profits with respect to $\tilde{q}$ equal to zero, we find that $\tilde{q} = 26,67$. The competing firm's profits are $\tilde{\pi} = 1.071$ and those of the other firm are \$867.

If both firms compete, Firm 1's profits are

$$\pi = (110 - q_1 - q_2)q_1 - 10q_1 - \frac{q_1^2}{2}$$

Setting the derivative of these profits with respect to q_1 equal to zero and using that $q_1 = q_2 = q_n$ in the symmetric equilibrium, we find that each firm produces $q_n = 25$, earning profits $\pi_n = \$937.5$. The diagram below represents these results as a normal-form game:

	<i>Collude</i>	<i>Compete</i>
<i>Collude</i>	1.000; 1.000	867; 1.071
<i>Compete</i>	1.071; 867	<u>937.5; 937.5.</u>

The outcome in which both firms compete is the unique Nash equilibrium of the game.

Question #4

Discuss the welfare consequences of a monopoly market structure. Use a graph to support your claims.

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Question #5

Please show that if the medium of exchange is continuous and if two firms have the same cost structure, ($c_2=c_1\equiv c$), then a Bertrand equilibrium is $p_1^B=p_2^B=c$.

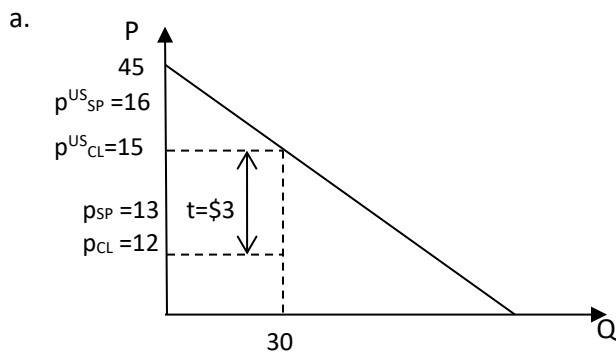
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Question #6

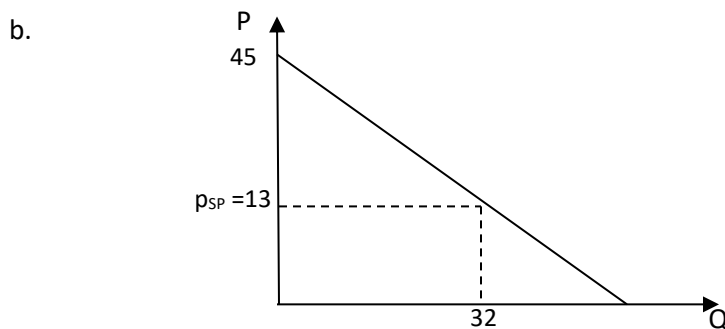
Consider the following world. There are three countries: United States (US), Chile (CL) and Spain (SP). Assume that US cannot affect the world prices. Only CL and SP produce red wine that is imported by US. Assume that red wine cannot be produced in US. In addition, consider that US's demand for imported red wine is $p^{US}=45-Q$, where Q denotes the quantity demanded and p^{US} is the domestic tariff-inclusive price.

- Consider the case in which US sets a uniform tariff of \$3 dollars per red wine bottle irrespective of where the wine is imported from. In addition, assume that the export price in CL is $p_{CL}=\$12$ and in SP is $p_{SP}=\$13$. Construct a graph indicating the US's import level under a uniform tariff and calculate the US's social welfare.
- Consider now the case in which US signs a free-trade agreement (FTA) with SP. Construct a graph representing the US's import under the FTA and calculate the new social welfare for US under this context.
- Compare and discuss your welfare results in part (a) and (b).

Solution



$$\text{Social Welfare (Tariff)} = CS + G = \frac{(45-15) \times 30}{2} + 3 \times 30 = 450 + 90 = 540$$



$$\text{Social Welfare (FTA)} = CS + G = \frac{(45-13) \times 32}{2} + 0 = 480 = 512$$

- Comparing our results we have that $SW(t) > SW(FTA)$, hence an increase on consumer surplus (from 450 to 480) does not compensate for the decrease in government revenue (from 90 to zero).