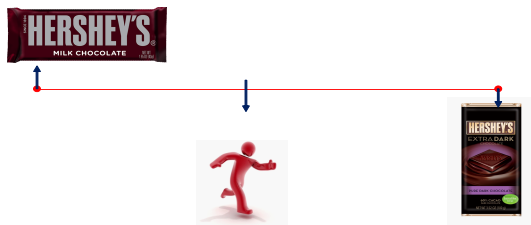


Location Models

Chapter 7.

- Consumers are heterogeneous
- Location can mean:
 - the physical location of the consumer (transportation cost)
 - Distance between the brand characteristic that a consumer views as ideal and the characteristic of the brand actually purchased.



- Horizontal differentiated products
 - Brands that are not uniformly utility ranked by all consumers

The Linear Approach (Hotelling 1929)

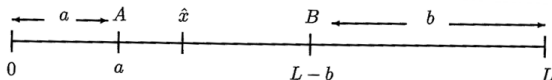
- Consumers reside on a linear street with a length of $L > 0$



- Consumers are uniformly distributed on this interval
- The total number of consumers is L
- Consumer is indexed by $x \in [0, L]$

Price game with fixed location

- Suppose two firms selling an identical product
- Assume that production is costless
- However these two firms are located in different points of the city



- Each consumer buys one unit
- τ is the transportation cost

Price game with fixed location

- Utility function of a consumer located at point x :

$$U_x = \begin{cases} -p_A - \tau |x - a| & \text{if he buys from A} \\ -p_B - \tau |x - (L - b)| & \text{if she buys from B} \end{cases}$$

- If $a < \hat{x} < L - b$, then

$$\begin{aligned} -p_A - \tau (\hat{x} - a) &= -p_B - \tau ((L - b) - \hat{x}) \\ \hat{x} &= \frac{p_B - p_A}{2\tau} + \frac{L - b + a}{2} \end{aligned}$$

- which is the demand function faced by firm A. The demand function faced by firm B is

$$L - \hat{x} = \frac{p_A - p_B}{2\tau} + \frac{L + b - a}{2}$$

Price game with differentiated products

- Bertrand-Nash equilibrium in price strategies

$$\max_{p_A} \pi_A = p_A \times \left[\frac{p_B - p_A}{2\tau} + \frac{L - b + a}{2} \right] \quad ((A))$$

$$\frac{\partial \pi_A}{\partial p_A} = \frac{p_B - 2p_A}{2\tau} + \frac{p_A(L - b + a)}{2} = 0 \quad / \times 2\tau$$

$$p_A = \frac{p_B + \tau(L - b + a)}{2} \quad (1)$$

Price game with differentiated products

- Let us think about p_A

$$p_A = \frac{p_B + \tau(L - b + a)}{2}$$



The distance between a and $(L - b)$ shrinks

- What happens if p_B increases?
- What happens if τ increases?

Price game with differentiated products

- Firm B takes p_A as given and choose p_B to

$$\max_{p_B} \pi_B = \frac{p_A p_B - p_B^2}{2\tau} + \frac{(L + b - a) p_B}{2} \quad ((B))$$

$$\frac{\partial \pi_B}{\partial p_B} = \frac{p_A - 2p_B}{2\tau} + \frac{(L + b - a)}{2} = 0$$

$$p_B = \frac{p_A + \tau(L + b - a)}{2} \quad (2)$$

- Substituting (2) into (1)

$$p_A = \frac{\left[\frac{p_A + \tau(L + b - a)}{2} \right] + \tau(L - b + a)}{2}$$

Price game with differentiated products

$$p_A = \frac{\left[\frac{p_A + \tau(L+b-a)}{2} \right] + \tau(L-b+a)}{2} \quad / \times 4$$

$$4p_A = p_A + \tau(L+b-a) + 2\tau(L-b+a)$$

$$3p_A = \tau[(L+b-a) + 2(L-b+a)]$$

$$3p_A = \tau[3L-b+a]$$

Hence, $p_A^h = \frac{\tau(3L-b+a)}{3}$ and $p_B^h = \frac{\tau(3L+b-a)}{3}$. $\hat{x}^h = \frac{(3L-b+a)}{6}$ and $\pi_A^h = \frac{\tau(3L-b+a)^2}{18}$

- Why these firms would like to make transportation as difficult as possible?
- Why Firms cannot be closely located?

Location and price game

- There is no solution when firm can choose price and location.
- What would firm A do, if given the price and location of firm B, it would be allowed to relocate

$$\frac{\partial \pi_A}{\partial a} > 0$$

- Principle of minimum differentiation
- **Proposition 7.7** *In the Hotelling linear-city game, there is no equilibrium for the game where firms use both prices and location as strategies*

Quadratic Transportation Cost



$$U_x \equiv \begin{cases} -p_A - \tau(x - a)^2 & \text{if he buys from A} \\ -p_B - \tau(x - (L - B))^2 & \text{if she buys from B} \end{cases}$$

- Solve a two-period game in which firms decide where to locate in the first period and set prices in the second period.
Max differentiation.

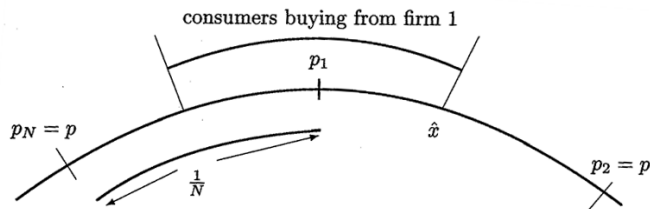
The circular approach

- An equilibrium exists when firms jointly decide on prices and location
- Let the city be the unit-circumference circle
- Firms:
 - Assume monopolistic competition market structure
 - N firms
 - Same technology
 - F fixed cost, c marginal cost and q_i output level of the firm-producing brand i

$$\pi_i(q_i) = \begin{cases} (p_i - c)q_i - F & \text{if } q_i > 0 \\ 0 & \text{if } q_i = 0 \end{cases}$$

The circular approach

- Consumers:
- Uniformly distributed on the unit circle
- τ is the transportation cost
- Each consumer buys one unit
- the distance between any two firms is $\frac{1}{N}$



The circular approach

- Consumer is indifferent:

$$\begin{aligned}p_1 + \tau \hat{x} &= p + \tau \left(\frac{1}{N} - \hat{x} \right) \\ \hat{x} &= \frac{p - p_1}{2\tau} + \frac{1}{2N}\end{aligned}$$

- Since firm 1 has a customer on its left and on its right, the demand function facing firm 1 is:

$$q_1(p_1, p) = 2\hat{x} = \frac{p - p_1}{\tau} + \frac{1}{N}$$

The circular approach

$$\begin{aligned}
 \max_{p_i} \pi_i(p_i, p^o) &= p_i q_i(p_i) - F - c q_i = (p_i - c) q_i - F \\
 &= (p_i - c) \left(\frac{p^o - p_i}{\tau} + \frac{1}{N} \right) - F \\
 \frac{\partial \pi_i}{\partial p_i} &= \frac{p^o - 2p_i + c}{\tau} + \frac{1}{N} = 0 \\
 \text{since } p_i &= p^o \text{ then } \frac{p - 2p + c}{\tau} + \frac{1}{N} = 0 \\
 p &= c + \frac{\tau}{N}
 \end{aligned}$$

The circular approach

- To find the equilibrium number of brands N , we set:

$$\pi_i(p^o, p^o) = (p^o - c)\left(\frac{1}{N}\right) - F = 0$$

$$\left(c + \frac{\tau}{N} - c\right)\left(\frac{1}{N}\right) - F = 0$$

$$\frac{\tau}{N^2} - F = 0 \quad / \times N^2$$

$$\tau - N^2 F = 0$$

$$N^2 = \frac{\tau}{F} \quad / \sqrt{\quad}$$

$$N = \sqrt{\frac{\tau}{F}}$$

The circular approach

Let us think about

$$N = \sqrt{\frac{\tau}{F}}$$

- Gas price



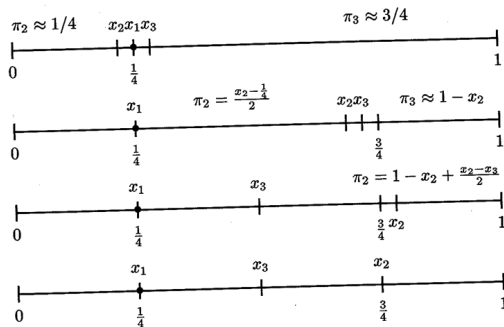
-



Sequential entry to linear city

- Prescott and Visscher (1977):
 - prices are fixed at a uniform level by the regulator
 - The only firm's choice variable is location
 - There are 3 firms entering sequentially
 - Firm 1 enters in period 1, Firm 2 in period 2 and firm 3 in period 3
 - SPE in location strategies, where each firm maximizes its market share
 - Location strategy by firm i : $0 \leq x_i \leq 1$, where $i = 1, 2, 3$
 - ϵ is the smallest possible measurable unit of distance
 - Assume **firm 1** has already moved and located itself at the point $x_1 = \frac{1}{4}$

Sequential entry to linear city



The Third-Period Subgame

- Firm 3 decides on its location x_3 after firm 1 and 2 are located
 - There are three possible location of firm 2
- ① $x_2 = \frac{1}{4} - \epsilon$: in this case firm 3 would locate at $x_2 = \frac{1}{4} + \epsilon$.
Here, $\pi_3 \approx \frac{3}{4}$ while $\pi_2 = x_2 + \frac{1}{2}(\frac{1}{4} - x_2) < \frac{1}{4}$

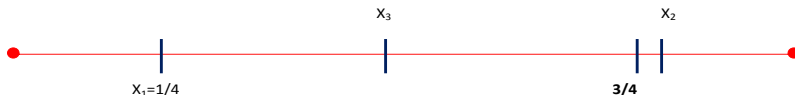


The Third-Period Subgame

2. $\frac{1}{4} < x_2 < \frac{3}{4}$: in this case firm 3 would locate to the right of firm 2, at $x_3 = x_2 + \epsilon$. Here, $\pi_3 \approx 1 - x_2$ while $\pi_2 \approx \frac{x_2 - \frac{1}{4}}{2} < \frac{1}{4}$. That is firm 2 shares the interval $[x_1, x_2]$

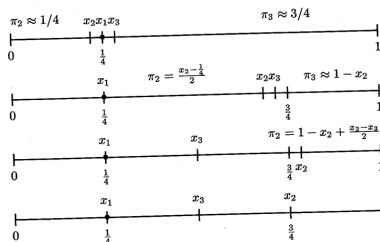


3. $x_2 \geq \frac{3}{4}$: in this case firm 3 would locate between firm 1 and 2, at any point $x_1 < x_3 < x_2$. Assume that $x_3 = \frac{\frac{1}{4} + x_2}{2}$, here $\pi_3 = \frac{x_2 - \frac{1}{4}}{2}$ and $\pi_2 = 1 - x_2 + \frac{x_2 - x_3}{2} = \frac{15 - 12x_2}{16}$



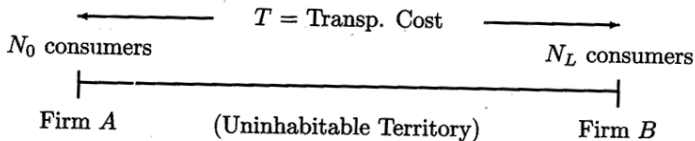
The Second-Period Subgame

- Why firm 2 locates at $x_2 \geq \frac{3}{4}$?
- In summary, the SPE is reached where:
 - $x_2 = \frac{3}{4}, \pi_2 = \frac{3}{8}, x_3 = \frac{1}{2}$ and $\pi_3 = \frac{1}{4}$



Calculus-Free location model

- Assume that production is costless
- Each consumer buys one unit
- Shopping on the other side of the town involves $T \geq 0$



Calculus-Free location model

- p_A denotes price charged by firm A and p_B the price charged by firm B
- Each consumer buys one unit
- Shopping on the other side of the town involves $T \geq 0$

$$U_0 \equiv \begin{cases} -p_A & \text{buying from } A \\ -p_B - T & \text{buying from } B \end{cases}$$

$$U_L \equiv \begin{cases} -p_A - T & \text{buying from } A \\ -p_B & \text{buying from } B \end{cases}$$

Calculus-Free location model



$$n_A \equiv \begin{cases} 0 & \text{if } p_A > p_B + T \\ N_0 & \text{if } p_B - T \leq p_A \leq p_B + T \\ N_0 + N_L & \text{if } p_A < p_B - T \end{cases}$$

$$n_B \equiv \begin{cases} 0 & \text{if } p_B > p_A + T \\ N_L & \text{if } p_A - T \leq p_B \leq p_A + T \\ N_0 + N_L & \text{if } p_B < p_A - T \end{cases}$$

Calculus-Free location model

- **Proposition 7.8:** There does not exist a Nash-Bertrand equilibrium in prices for the discrete version of Hotelling's location Model
- **Proof:**
- By way of contradiction, suppose that $\{P_N^A, P_N^B\}$ constitutes a NE. Three cases (1) $|P_N^A - P_N^B| > T$;
(2) $|P_N^A - P_N^B| < T$; (3) $|P_N^A - P_N^B| = T$.

•

