

Homework #2 (Due on October 27th, 2026)

1. Two firms have different technologies for producing identical erasers. Assume that all erasers are sold in boxes containing 100 of them. Firm A can produce each box at a unit cost of $c_A = \$6$ whereas firm B (less efficient) at a unit cost of $c_B = \$8$.
 - a. Suppose that the aggregate market demand for boxes of erasers is $p = 12 - Q/2$, where p is the price per box and Q is the number of boxes sold. Solve for the Nash-Bertrand equilibrium prices p_A^b and p_B^b , and the equilibrium profits Π_A^b and Π_B^b .

Solution

The first case to be checked is where the efficient firm A undercuts B by setting $p_A = c_B - \varepsilon = 8 - \varepsilon$, where ε is a small number. Firm B set $p_B = c_B = 8$. In this case, all consumers buy brand A only, hence, solving $8 = 12 - q_A / 2$ yields $q_A = 8$ and $q_B = 0$. The profits are then $\pi_A = (8 - 6) \times 8 = 16$ and $\pi_B = 0$.

The second case to be checked is where A sets a monopoly price. Solving $MR = 12 - q_A = c_A = 6$ yields $q_A = 6$.

Hence $p_A = 12 - 6 / 2 = \$9 > \$8 = p_B$. Therefore, in this case, firm A cannot charge its monopoly price. Altogether, a Nash-Bertrand equilibrium prices are $p_A = 8 - \varepsilon$ and $p_B = 8$. The equilibrium profits are therefore $\pi_A = (8 - 6) \times 8 = 16$ and $\pi_B = 0$.

2. The demand function for concert played by the Valencia symphony orchestra varies between nonstudents (N) and students (S). Formally, the two inverse demand functions for these consumer groups are given by

$$p_N = 12 - q_N \text{ and } p_S = 6 - q_S.$$

Thus, at any given consumption level nonstudents are willing to pay a higher price than students. Assume that the orchestra's total cost function is $C(Q) = 10 + 2Q$ where $Q = q_N + q_S$ is to total number of tickets sold. Solve the following questions.

- a) Suppose the orchestra is able to price discriminate between the two consumer groups by asking students to present their student ID cards to be eligible for a student discount. Compute the profit-maximizing prices p_N and p_S , the number of tickets sold to each group of consumers, and total monopoly profit.

- b) Suppose now the local mafia has distributed a large number of fake student ID cards, so basically every resident has a student ID card regardless of whether the resident is a student or not. Compute the profit-maximizing price, the number of tickets sold to each group of consumers, and total profit assuming that the monopoly orchestra is unable to price discriminate.
- c) By how much the orchestra enhances its profit from the introduction of student discounted tickets compared with the profit generated from selling a single uniform ticket price to both consumer groups.

Solution

- a) Equating the marginal revenue to marginal cost in the market for nonstudents yields $MR_N = 12 - 2q_N = 2$, hence, $q_N = 5$. Therefore, $p_N = 12 - 5 = \$7$. Similarly, in the market for students $MRS = 6 - 2q_S = 2$, hence, $q_S = 2$. Therefore, $p_S = 6 - 2 = \$4 < p_N$. Thus, students indeed receive a discount of $p_N - p_S = 7 - 4 = \$3$.

Next, combined total profit from selling in both markets is $\pi^D = (7-2)5 + (4-2)2 = 10 = \19 .

- b) We first check how much profit can be earned if the seller sets a sufficiently low price so the entire market is served. To compute this, we first must find the aggregate market demand curve. Inverting the two demand curves yield $q_N = 12 - p$ and $q_S = 6 - p$. The aggregate demand curve is then $Q = q_N + q_S = 18 - 2p$. The resulting inverse aggregate demand function and the marginal revenue function are

$$p = \frac{18 - Q}{2} \text{ and } MR = 9 - Q.$$

Solving $MR=9-Q=2=MC$ yields $Q=7$, therefore, $p=(18-7)/2=11/2 < 6$. Altogether, the profit when both markets are served is

$$\pi^{both} = \left(\frac{11}{2} - 2\right)7 - 10 = \frac{29}{2} = 14.5.$$

We are now able to compute the profit assuming that the monopoly sets a sufficiently high price so only nonstudents can “afford” to purchase concert tickets. Thus, under a sufficiently high price, $q_S = 0$, and the monopoly solves $MR_N(q_N) = 12 - 2q_N = 2$ yielding $q_N = 5$. The price should set to $p = 12 - 5 = \$7$. Actually, we have already calculated these figures in the analysis of price discrimination above. What is important to check is that $p = 7 > 6$ which is the intercept of the students' inverse demand function. Hence, students don't buy at this price. Total profit is then given by

$$\pi^{ND} = \pi^{1 \text{ only}} = (7 - 2)5 - 10 = 15 > 14.5.$$

Since $\pi^{1 \text{ only}} > \pi^{both}$, the monopoly earns a higher profit when setting a sufficiently high price so only nonstudents purchase concert tickets.

- c) The gain from price discrimination is therefore $\pi^D - \pi^{1 \text{ only}} = 19 - 15 = 4$

3. In Valencia there are two suppliers of distilled water, labeled as firm Aqua (A) and firm Blue (B). Distilled water is considered a homogenous good. Let p denote the price per

gallon, q_A quantity sold by firm Aqua, and q_B the quantity sold by firm Blue. Both firms are located close to a spring so the only production cost is the cost of bottling. Formally, each firm bears a production cost of $c_A = c_B = \$3$ per one gallon of water.

Valencia's aggregate inverse demand function for distilled water is given by $p = 12 - Q = 12 - q_A - q_B$, where $Q = q_A + q_B$ denotes the aggregate industry supply of distilled water in Valencia. Solve the following problems:

- Solve for firm A's best-response function, $q_A = R_A(q_B)$. Also solve for firm B's best-response function, $q_B = R_B(q_A)$. Show your derivations.
- Solve for the Cournot equilibrium output levels q_A^c and q_B^c . State which firm sells more water (if any) and why.
- Solve for the aggregate industry supply and the equilibrium price of distilled water in Valencia.
- Solve for the profit level made by each firm, and for the aggregate industry profit. Which firm earns a higher profit and why?

Solution:

- The best-response functions are given by

$$q_A(q_B) = (9 - q_B)/2 \text{ and } q_B(q_A) = (9 - q_A)/2$$
- The above best-response functions constitute two linear equations with two variables, q_A and q_B . The unique solution is $q_A^c = q_B^c = 3$ gallons. Both firms produce the same amount since they are equally efficient in the sense that they bear identical production costs.
- $Q = q_A + q_B = 3 + 3 = 6$ gallons. The equilibrium price is $p = 12 - Q = \$6$.
- Since there are no fixed costs, $\pi_A = (p - c_A)q_A = (6 - 3)3 = \9 . Similarly, $\pi_B = (p - c_B)q_B = (6 - 3)3 = \9 . Industry profit is then $\Pi = \pi_A + \pi_B = 9 + 9 = \18 . Both firms earn the same profit since they bear identical production costs.