

GEEO 36158 - Homework #1 (Due on October 1st, 2026)

Question #1 (2 points)

The table below displays the profits from a price game between Apple and Microsoft.

		Microsoft	
Apple		<i>Low Price</i>	<i>High Price</i>
	<i>Low</i>	(50,50)	(190,0)
	<i>High</i>	(0,425)	(100,150)

Each firm can set either a high price, p_H , or a low price, p_L , where $p_H > p_L > 0$. Solve the following problems.

- Write down Apple's best-response function. Also, write down Microsoft's best-response function.
- Conclude which outcomes of the game constitute a Nash equilibrium.
- Are there outcomes in this game which Pareto dominate the Nash equilibrium outcomes that you found in the previous problem? Prove your result.
- Suppose Apple and Microsoft form a cartel. Which outcome maximizes joint industry profit?

Solution:

- a) The firms' best-response functions are given by:

$$p_A = R_A(p_M) = \begin{cases} p^L & \text{if } p_M = p^H \\ p^L & \text{if } p_M = p^L \end{cases} \text{ and } p_M = R_M(p_A) = \begin{cases} p^L & \text{if } p_A = p^H \\ p^L & \text{if } p_A = p^L \end{cases}$$

Remark: Firm Apple's best-response function implies that p^L is a dominant action. Similarly, p^L is a dominant action for Microsoft.

- The two best-response function "intersect" at $\{p^A, p^M\} = \{p^L, p^L\}$. Hence, the outcome $\{p^L, p^L\}$ constitutes a unique Nash equilibrium.
- Yes, the outcome $\{p^H, p^H\}$. Pareto dominates $\{p^L, p^L\}$. since $\Pi^A(p^H, p^H) = 100 > 50 = \Pi^A(p^L, p^L)$, and $\Pi^M(p^H, p^H) = 150 > 50 = \Pi^M(p^L, p^L)$
- Industry profit is maximized when Apple sets $p^A = p^H$ and Microsoft sets $p^M = p^L$, in which case $\Pi^G + \Pi^F = 425 + 0 = 425$.

Question #2 (2 points)

Firms A and B can choose to adopt a new technology (N) or to adhere to their old technology (O). Formally, firms' action sets are: $t_A \in \{N, O\}$ and $t_B \in \{N, O\}$. The table below exhibits the profit made by each firm under different technology choices.

		Firm B	
		<i>New Technology</i>	<i>Old Technology</i>
Firm A	<i>New</i>	(200,0)	(0,200)
	<i>Old</i>	(50,100)	(100,50)

- Write down the best-response functions of firms A and B, $t_A = R_A(t_B)$ and $t_B = R_B(t_A)$
- Draw the tree of a two-stage extensive-form game in which firm A chooses its technology t_A in stage I, and Firm B chooses its t_B in stage II (after observing the choice made by firm A). Make sure that you indicate firms' profits at the termination points on the tree. Solve for the *subgame-perfect equilibrium* of this game. Provide a short proof or an explanation justifying your answer.
- Draw the tree of a two-stage extensive-form game in which firm B chooses its technology t_B in stage I and Firm A chooses its t_A in stage II (after observing the choice made by firm B). Solve for the *subgame-perfect equilibrium* of this game. Provide a short proof or an explanation justifying your answer.
- Compare the equilibrium firms' profit levels of the games played in (b) and in (c). Conclude under which game firm A earns a higher profit. Briefly explain your answer.

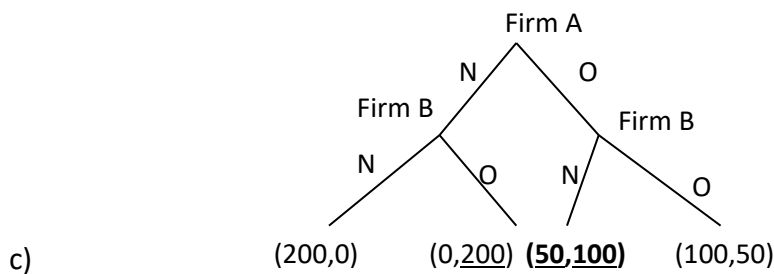
Solution

$$t_A = BR_A(t_B) = \begin{cases} N & \text{if } t_B = N \\ O & \text{if } t_B = O \end{cases} \text{ and } t_B = BR_B(t_A) = \begin{cases} O & \text{if } t_A = N \\ N & \text{if } t_A = O \end{cases}$$

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- From firm B's best response function given above, if $t_A = N$, $t_B(N) = O$, in which case firm A earns $\pi^A(N,O) = 0$. Instead, if $t_A = O$, $t_B(O) = N$, in which case firm A earns $\pi^A(O,N) = 50 > 0$. Therefore, the subgame-perfect equilibrium for this game is:

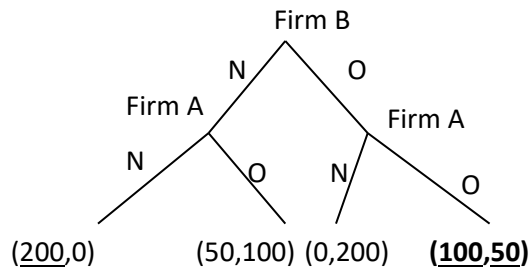
$$t_A = O \text{ and } t_B = BR_B(t_A) = \begin{cases} O & \text{if } t_A = N \\ N & \text{if } t_A = O \end{cases}$$

Note: Although there is no Nash equilibrium of the normal-form game a SPE of extensive-form game does exist, because the extensive-form game is somewhat different than the normal-form game.



From firm A's best response function given above, if $t_B = N$, $t_A(N) = N$, in which case firm B earns $\Pi^B(N,N) = 0$. Instead, if $t_B = O$, $t_A(O) = O$, in which case firm B earns $\Pi^B(O,O) = 50 > 0$. Therefore, the subgame-perfect equilibrium for this game is:

$$t_B = O \text{ and } t_A = BR_A(t_B) = \begin{cases} N & \text{if } t_B = N \\ O & \text{if } t_B = O \end{cases}$$



d) In the game (ii) $\Pi^A(O,N) = 50$ and $\Pi^B(O,N) = 100$. In the game (iii) $\Pi^A(O;O) = 100 > 50$ and $\Pi^B(O,O) = 50$. This game is interesting because A has a first-mover disadvantage. Not every game yields this result. This is because A would like to “match” B's technology, whereas B gains from introducing a different technology. By letting B making the first choice, A is able to match its technology choice with B's choice.

Question #3 (1.5 points)

Emma and Oscar are deciding whether to engage in outdoor or indoor activities. Assume that both players value being together, but Oscar values it more than Emma.

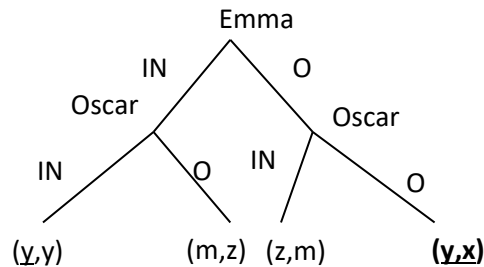
- Model this situation as a simultaneous-move game.
- Then, model it as a sequential-move game, where Emma moves first.
- Clearly state your assumptions about payoffs (you may use general parameters like x or y).
- Discuss the equilibrium results in each time structure and compare your results.

Solution

a)

		<i>Emma</i>	
		<i>Outdoor</i>	<i>Indoor</i>
<i>Oscar</i>	<i>Outdoor</i>	(x,y)	(z,m)
	<i>Indoor</i>	(m,z)	(y,y)

b)



Question #4 (1 point)

Consider the cost function $TC(Q) \equiv F + cQ$, where $F, c > 0$.

- Calculate and plot the $TC(Q)$, $AC(Q)$ and $MC(Q)$.
- At what output level is the average cost minimized?
- Infer whether this technology exhibits IRS, CRS, or DRS. Explain.

Solution

- $TC(Q) = F + cQ$, $AC(Q) \equiv F/Q + c$ and $MC(Q) = c$ (Figure 4.2 Textbook)
- Clearly, $AC(Q)$ declines with Q . Hence, $AC(Q)$ is minimized when $Q = +\infty$.
- Declining average cost function reflects an increasing return to scale technology.

Question #5 (2 points)

In Seattle there is only one shoe repair factory who acts as a monopoly. The inverse demand function for this service is given by $P = 12 - \frac{Q}{2}$, where P denotes the price charged per visit, and Q the quantity demanded for shoe repair.

- Suppose the cost function of this shoe repair factory is given by $C(Q) = 4 + 2Q$. That is, the marginal cost is $c = \$2$ (consisting of her value time and other "communication" expenses), and the fixed cost is $F = \$4$ (say, monthly rent on her office space). Compute and draw the shoe factory's marginal cost and average functions, as well as the marginal revenue function.
- Algebraically compute the shoe factory's profit-maximizing output, price, and profit.
- Compute the price elasticity at the profit-maximizing output.

Solution

- The marginal and average cost functions are given by:

$$MC(Q) = \frac{\partial C(Q)}{\partial Q} = 2 \text{ and } AC(Q) = \frac{C(Q)}{Q} = \frac{4}{Q} + 2$$

The marginal revenue function is $MR(Q) = 12 - Q$

- The monopoly equates $MR(Q) = 12 - Q = 2 = MC(Q)$ to obtain the profit-maximizing output level $Q = 10$. The monopoly price is then $p = 12 - Q/2 = \$7$. Finally, the profit is $\pi = 7 \times 10 - 2 \times 10 - 4 = \46 .

- c) The direct demand function is $Q(p) = 24 - 2p$. Then, the price elasticity is:

$$\epsilon_p = \frac{\partial Q}{\partial p} \times \frac{P}{Q} = -2 \frac{7}{10} = -1.4$$

Thus, the demand is elastic.

Question #6 (1.5 points)

Historically, most of the diamond mines in the world have been controlled by a few companies and governments. Through clever marketing by diamond producers, many consumers have furthermore become convinced that “diamonds are a girl’s best friend” because “diamonds are forever.” Suppose that the demand for diamond is given by $p=90-x$ and suppose further that the monopolist’s marginal cost curve is given by $MC=x$.

- Derive the equation for the marginal revenue curve
- What is the profit-maximizing output level x^M ? What is the profit-maximizing price p^M (assuming that the monopolist can only charge a single per-unit price to all consumers)?
- In absence of recurring fixed costs, what is the monopolist’s profit?
- What is the consumer surplus and deadweight loss (assuming that demand is equal to marginal willingness to pay)?
- What is the cost function if recurring fixed costs are sufficiently high to cause the monopolist’s profit to be zero?

Solution

- $MR(x)=90-2x$
- $MR(x)=MC(x)$, hence $90-2x=x$, then $x^M=30$ and $P^M=90-30=60$
- Monopolist’s Profit: $60 \cdot 30 - 30^2/2 = 1,800 - 450 = 1,350$. Note that Total Cost is $X^2/2$ (the integral of the marginal cost)
- $CS=(90-60) \cdot 30/2 = 450$ note that $DWL=(60-30) \cdot (45-30)/2 = 225$
- The total cost: $F + X^2/2$, where F is fixed cost and $F=1,350$