

Exercise – Hotelling

University Road is best described as the interval $[0,1]$. Two fast-food restaurants serving identical food are located at the edges of the road, so that restaurant 1 is located on the most left-hand side, and restaurant 2 is located on the most right-hand side of the road. Consumers are uniformly distributed on the interval $[0,1]$, where at each point on the interval lives one consumer. Each consumer buys one meal from the restaurant in which the price plus the transportation cost is the lowest. In University Road, the wind blows from right to left, hence the transportation cost for a consumer who travels to the right is $\$R$ per unit of distance, and only $\$1$ per unit of distance for a consumer who travels to the left. Answer the following questions:

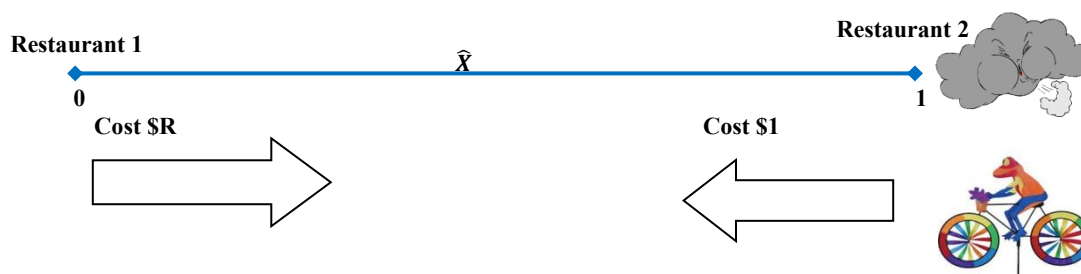
- (a) Let p_i denote price of meal at restaurant i , $i=1,2$. Assume that p_1 and p_2 are given and satisfy

$$0 < p_1 - R < p_2 < 1 + p_1$$

Denote by $\hat{x}$ the location of the consumer who is indifferent to whether he or she eats at restaurant 1 or restaurant 2 and calculate $\hat{x}$ as a function of p_1 , p_2 , and R .

- (b) Suppose that the given prices satisfy $p_1 = p_2$. What is the minimal value of the parameter R such that all consumers will go to eat only at restaurant 1?

Solution [Part a]



The consumer is indifferent between eating in restaurant 1 or restaurant 2 when his/her utility:

$$U_x(\text{Restaurant 1}) = U_x(\text{Restaurant 2})$$

$$-p_1 - 1 \times \hat{x} = -p_2 - R \times (1 - \hat{x})$$

Hence solving for $\hat{x}$, we obtain,

$$\hat{x} = \frac{p_2 - p_1 + R}{1 + R}$$

Therefore, restaurant 1 faces a demand $\hat{x}$, and restaurant 2 faces a demand $1 - \hat{x}$, that is

$$1 - \hat{x} = 1 - \frac{p_2 - p_1 + R}{1 + R} = \frac{1 - p_2 + p_1}{1 + R}$$

Restaurant 1's maximization problem is

$$\max \pi_1 = \hat{X} \times p_1 = \frac{p_2 - p_1 + R}{1 + R} \times p_1$$

$$\frac{\partial \pi_1}{\partial p_1} = \frac{p_2 - 2p_1 + R}{1 + R} = 0$$

Solving for p_1 we obtain that (A) $p_1 = \frac{p_2 + R}{2}$. Restaurant 2's maximization problem is

$$\max \pi_2 = (1 - \hat{X}) \times p_2 = \frac{1 - p_2 + p_1}{1 + R} \times p_2$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{1 - 2p_2 + p_1}{1 + R} = 0$$

Solving for p_2 we obtain that (B) $p_2 = \frac{p_1 + 1}{2}$. Finally, substituting (B) into (A):

$$p_1 = \frac{\left(\frac{p_1 + 1}{2}\right) + R}{2} = \frac{p_1 + 1}{4} + \frac{R}{2}$$

Hence, $p_1 = \frac{1 + 2R}{3}$ and substituting p_1 into (B) we have that $p_2 = \frac{\left(\frac{1 + 2R}{3}\right) + 1}{2}$ or $p_2 = \frac{2 + R}{3}$. In addition, we can obtain restaurant 1's and 2's profits:

$$\pi_1 = \hat{X} \times p_1 = \frac{\frac{2 + R}{3} - \frac{1 + 2R}{3} + R}{1 + R} \times \frac{1 + 2R}{3}$$

$$\pi_2 = (1 - \hat{X}) \times p_2 = \frac{1 - \frac{2 + R}{3} + \frac{1 + 2R}{3}}{1 + R} \times \frac{2 + R}{3}$$