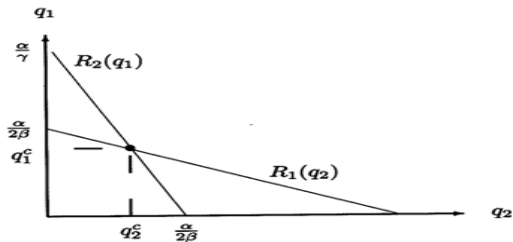


Markets for Differentiated Products

Chapter 7.



Simple Models

- Most industries produce large number of similar but not identical products
- Industries are concentrated



2015 Nissan LEAF 2015 Nissan Xterra



Example: Designer Water

Brand	\$ per 1/2-Liter Bottle	Source	1997 sales
Aqua Fina	\$0.59 - \$0.69	Purified tap water	\$60 million
Crystal Geyser	\$1.00	Springs in CA & TN	\$115 million
Evian	\$0.99 - \$1.09	Spring French Alps	\$185 million
Poland spring	\$1.39	Spring in Maine	\$300 million

Simple Models

- Consumers buy a small subset
- Consider a two-firm industry producing two differentiated products ($i = 1, 2$)
- Production is *costless*
- The Inverse Demand structure:

$$p_1 = \alpha - \beta q_1 - \gamma q_2 \text{ and } p_2 = \alpha - \gamma q_1 - \beta q_2 \text{ where } \beta > 0, \beta^2 > \gamma^2$$

Simple Models



$$p_1 = \alpha - \beta q_1 - \gamma q_2 \text{ and } p_2 = \alpha - \gamma q_1 - \beta q_2 \text{ where } \beta > 0, \beta^2 > \gamma^2$$

- note that the assumption $\beta^2 > \gamma^2$ is very important!
- The *own-price effect* dominates the *cross-price effect*:
 - The price of a brand is more sensitive to a change in the quantity of this brand than to a change in the quantity of the competing brand.
- Measure of differentiation $\delta = \frac{\gamma^2}{\beta^2}$:
 - if $\gamma^2 \rightarrow 0$ ($\delta \rightarrow 0$) Highly differentiated
 - if $\gamma^2 \rightarrow \beta^2$ ($\delta \rightarrow 1$) almost homogeneous

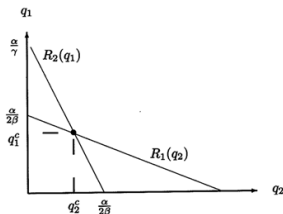
Quantity game with differentiated products



$$\max_{q_i} \pi_i(q_1, q_2) = (\alpha - \beta q_i - \gamma q_j) q_i \quad i, j = 1, 2, \quad i \neq j$$

$$\frac{\partial \pi_i}{\partial q_i} = \alpha - 2\beta q_i - \gamma q_j = 0$$

$$q_i = \frac{\alpha - \gamma q_j}{2\beta} \quad i, j = 1, 2, \quad i \neq j$$



Quantity game with differentiated products



$$q_i = \frac{\alpha - \gamma q_j}{2\beta} \text{ and } q_j = \frac{\alpha - \gamma q_i}{2\beta}$$



$$q_i^c = \frac{\alpha}{2\beta + \gamma}$$

$$p_i^c = \frac{\alpha\beta}{2\beta + \gamma}$$

$$\pi_i^c = \frac{\alpha^2\beta}{(2\beta + \gamma)^2}$$

- **Proposition:** *In a Cournot game with differentiated products, the profits of firms increase when the products become more differentiated.*

Price game with differentiated products

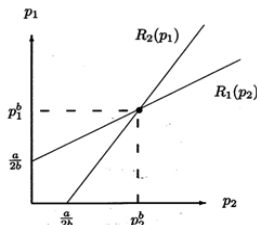


$$\max_{p_i} \pi_i(p_1, p_2) = (a - bp_i + cp_j)p_i \quad i, j = 1, 2, \quad i \neq j$$

$$\frac{\partial \pi_i}{\partial p_i} = a - 2bp_i + cp_j = 0$$

$$p_i = \frac{a + cp_j}{2b} \quad i, j = 1, 2, \quad i \neq j$$

- Note that $a \equiv \frac{\alpha(\beta-\gamma)}{\beta^2-\gamma^2}$, $b \equiv \frac{\beta}{\beta^2-\gamma^2}$, $c \equiv \frac{\gamma}{\beta^2-\gamma^2}$



Price game with differentiated products

- Strategic substitutes if the BRFs are downward sloping
- Strategic complements if the BRFs are upward sloping
- $$p_i^C - p_i^B = \frac{\alpha\beta}{2\beta+\gamma} - \frac{\alpha(\beta-\gamma)}{2\beta-\gamma} = \frac{\alpha}{4\frac{\beta^2}{\gamma^2}-1}$$
- In a differentiated products industry:
 - $p_i^C > p_i^B$
 - The more differentiated the products are, the smaller the difference
 - This difference is zero when products become independent.

Sequential-moves price game

- Assume $q_1 = 168 - 2p_1 + p_2$ and $q_2 = 168 + p_1 - 2p_2$
- The single-period game Bertrand prices and profits are:
 $p_i^B = 56$ and $\pi_i^B = 6,272$
- However, in the sequential game:

$$\begin{aligned}\max_{p_1} \pi_1(p_1, R_2(p_1)) &= (168 - 2p_1 + \frac{168 + p_1}{4})p_1 \\ \frac{\partial \pi_1}{\partial p_1} &= 210 - \frac{7}{2}p_1\end{aligned}$$

- $p_1^S = 60, p_2^S = 57, q_1^S = 105, q_2^S = 114, \pi_1^S = 6,300 > \pi_1^B$
and $\pi_2^S = 6,498 > \pi_2^B$
- Under a sequential-move price game:
 - $\pi_i^S > \pi_i^B$
 - The leader makes a lower profit than the follower

Monopolistic Competition in Differentiated Products

- Objective: Calculate the equilibrium number of differentiated brand produced by the industry
- Consumer are homogeneous
- Unlimited number of potentially produced brands
- Free entry

Model

- Dixit and Stiglitz 1977
- An industry producing differentiated brands $i = 1, 2, \dots, N$
- q_i quantity produced/consumed of brand i , p_i the price of brand i
- Consumers: love-for-variety property
- $u(q_1, q_2, \dots, q_N) \equiv \sum_{i=1}^N \sqrt{q_i}$
- Love for variety: $\lim_{q_i \rightarrow 0} \frac{\partial u_i}{\partial q_i} = \lim_{q_i \rightarrow 0} \frac{1}{2\sqrt{q_i}} = +\infty$
- Consumer's income: $I \equiv L + \sum_{i=1}^N \sqrt{\pi_i(q_i)}$
- **Budget constraint:** $\sum_{i=1}^N p_i q_i \leq I$

Model

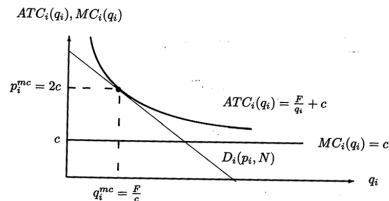
- Lagrangian: $L(q_i, p_i, \lambda) \equiv \sum_{i=1}^N \sqrt{q_i} + \lambda \left[I - \sum_{i=1}^N p_i q_i \right]$
- The first-order condition for every brand i is:

$$\frac{\partial L(q_i, p_i, \lambda)}{\partial q_i} = \frac{1}{2\sqrt{q_i}} - \lambda p_i$$

- Therefore: the direct demand, inverse demand and price elasticity are:
 - $q_i(p_i) = \frac{1}{4\lambda^2(p_i)^2}$, $p_i(q_i) = \frac{1}{2\lambda\sqrt{q_i}}$, $\eta \equiv \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i} = -2$
 - Why an increase in price would not have an effect on consumers' expenditure

Brand-producing firms

- Each brand is produced by a single firm
- identical cost structure (IRS)
- $TC_i = F + cq_i$ if $q_i > 0$
-



- What is the effect of N on the demand
- *Chamberlin's tangency condition*
- How many Brands Will be produced in this economy?