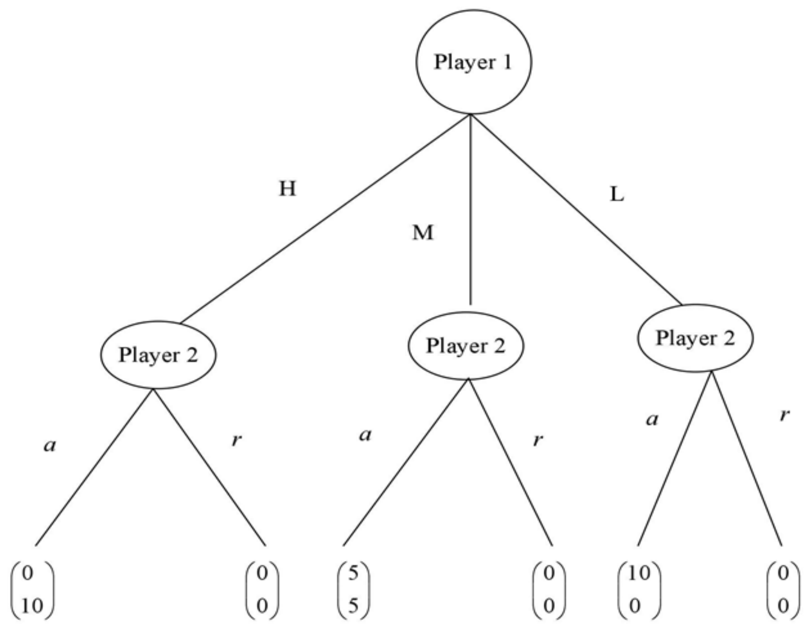
# Homework # 5 [Due on Tuesday April 14th, 2026]

1. Find a story that can be represented by an extensive form game. Identify the: (i) set of players, (ii) set of actions, (iii) time structure of the game and (iv) payoffs.

   - For instance, the example of "Antz vs A Bug's life"

2. Consider the following extensive form game.




   (a) What are the strategies for player 1?
      - The strategies for player 1 are $H$, $M$, and $L$.

   (b) What are the strategies for player 2?
      - The strategies for player 2 are: $\{aaa, aar, arr, rrr, rra, raa, ara, rar\}$

   (c) Take your results from a) and b) and construct a matrix representing its normal form game representation.
      -

Player 2

| | | $aaa$ | $aar$ | $arr$ | $rrr$ | $rra$ | $raa$ | $ara$ | $rar$ |
|---|---|---|---|---|---|---|---|---|---|
| Player 1 | $H$ | 0, 10 | 0, 10 | 0, 10 | 0,0 | 0,0 | 0,0 | 0,10 | 0,0 |
| | $M$ | 5, 5 | 5, 5 | 5, 5 | 0,0 | 0,0 | 5,5 | 0,0 | 5,5 |
| | $L$ | 10, 0 | 10, 0 | 10, 0 | 0,0 | 10,0 | 10,0 | 10,0 | 0,0 |

3. Consider the following normal form game

| | | Player 2: L | Player 2: R |
|---|---|---|---|
| Player 1 | U | -10, 10 | 0, 12 |
| | C | -12, 0 | 2, 2 |
| | D | -9, 0 | 1, 0 |

(a) Find strictly dominant strategies (if any) for player 1 and for player 2.

- There are no strictly dominant strategies for both players 1 and 2.

(b) Find strictly dominated strategies (if any) for player 1 and for player 2.

- **For player 1**, strategy $U$ is strictly dominated because when player 2 chooses strategy $L$, he could maximize his payoff by choosing strategy $D$, and similarly, by choosing strategy $C$ when player 2 chooses strategy $R$. **For player 2**, neither strategy $L$ nor $R$ is strictly dominated because when player 1 chooses strategy $D$, player 2 obtains the same payoff by either choosing strategy $L$ or $R$, both of which give him exactly the same payoff of 0.

(c) If you apply iterative deletion of strictly dominated strategies (IDSDS), what is the surviving strategy pair (or pairs)? Explain the steps you use in IDSDS, and why you use them.

- Since strategy $U$ is strictly dominated, we can remove this strategy for player 1, resulting in a reduced-form matrix as follows:

| | | Player 2: L | Player 2: R |
|---|---|---|---|
| Player 1 | C | -12, 0 | 2, 2 |
| | D | -9, 0 | 1, 0 |

At this stage of the game, none of the strategies are strictly dominated by one another, and therefore, the strategy profile that survives IDSDS becomes

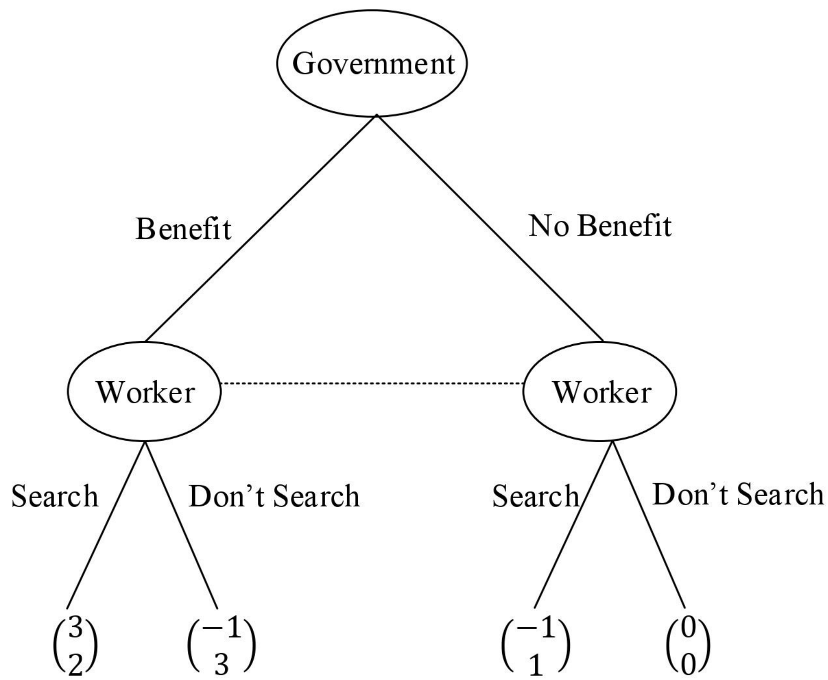
$$\{(C, L), (C, R), (D, L), (D, R)\}$$

4. Consider the following simultaneous-move game between the government (row player), which decides whether to offer unemployment benefits, and an unemployed worker (column player), who chooses whether to search for a job. As you interpret from the payoff matrix below, the unemployed worker only finds it optimal to search for a job

when he receives no unemployment benefit; while the government only finds it optimal to help the worker when he searches for a job.

| | | Worker | |
|---|---|---|---|
| | | *Search* | *Don't Search* |
| Government | *Benefit* | 3, 2 | -1, 3 |
| | *No benefit* | -1, 1 | 0, 0 |

(a) Represent this game in its extensive form (game tree), where the government acts first and the worker responds without observing whether the government offered unemployment benefits.

- 


(b) Does government has strictly dominant strategies? How about the worker?

- There are no strictly dominant strategies for both government and worker. Specifically, when the worker chooses (not) to search for a job, the government will be better off (not) offering unemployment benefits. Whereas, when the government chooses (not) to offer unemployment benefits, the worker will be better off (not) searching for a job.

(c) Find which strategy profile (or profiles) survive the application of IDSDS.

- In this context, every strategy survives IDSDS, which contribute to a strategy profile of

$$\{(Benefit, Search), (Benefit, Don'tSearch), (NoBenefit, Search), (NoBenefit, Don'tSearch)\}$$

5. Consider the following simultaneous-move game played by player 1 (in rows) and player 2 (in columns).

*Player 2*

| Player 1 | | $x$ | $y$ | $z$ |
|---|---|---|---|---|
| | $a$ | 2, 3 | 1, 4 | 3, 2 |
| | $b$ | 5, 1 | 2, 3 | 1, 2 |
| | $c$ | 3, 7 | 4, 6 | 5, 4 |
| | $d$ | 4, 2 | 1, 3 | 6, 1 |

(a) Which strategy pairs survive the application of iterative deletion of strictly dominated strategies (IDSDS)?

- For player 2 (column player), strategy $z$ is strictly dominated by $y$. We can then delete column $z$, leaving us with the following reduced-form matrix.

*Player 2*

| Player 1 | | $x$ | $y$ |
|---|---|---|---|
| | $a$ | 2, 3 | 1, 4 |
| | $b$ | 5, 1 | 2, 3 |
| | $c$ | 3, 7 | 4, 6 |
| | $d$ | 4, 2 | 1, 3 |

For player 1 (row player), strategy $a$ is strictly dominated by $b$. After deleting the row corresponding to $a$, we obtain

*Player 2*

| Player 1 | | $x$ | $y$ |
|---|---|---|---|
| | $b$ | 5, 1 | 2, 3 |
| | $c$ | 3, 7 | 4, 6 |
| | $d$ | 4, 2 | 1, 3 |

At this point, we cannot delete any more strategies for players 1 or 2 if we restrict them to use pure strategies. However, if we allow player 1 to randomize between the strategies that provide the highest payoff, $b$ and $c$. In particular, assigning a probability $p$ to strategy $b$ and the remaining probability $1 - p$ to strategy $c$, player 1's expected payoff when player 2 chooses strategy $x$ (in the left-hand column of the above matrix) is

$$5p + 3(1 - p) = 2p + 3$$

which is larger than player 1's payoff from strategy $d$, 4, as long as $2p+3 > 4$, or solving for $p$, if $p > \frac{1}{2}$. Similarly, when player 2 chooses straegy $y$ (in the right-hand column of the above matrix), player 1's expected payoff from randomizing between $b$ and $c$ becomes

$$2p + 4(1 - p) = 4 - 2p$$

which is larger than player 1's payoff from strategy $d$, 1, as long as $4-2p > 1$, or solving for $p$, if $p < \frac{3}{2}$. This condition holds by assumption since probability $p$ must be a number between 0 and 1. Therefore, any randomization between strategies $b$ and $c$ that assigns more than 50% probability on strategy $b$ (that is, $p > 1/2$) yields a expected utility larger than the utility player 1 receives from strategy $d$. We can therefore claim that strategy $d$ is strictly dominated, and delete the bottom row of the above matrix, leaving us with the followed reduced-form matrix.

| | | *Player 2* | |
|---|---|---|---|
| | | $x$ | $y$ |
| *Player 1* | $b$ | $5,1$ | $2,3$ |
| | $c$ | $3,7$ | $4,6$ |

At this point, we cannot delete any further strategies for players 1 or 2. Then, the strategy profiles surviving IDSDS are those in the four cells of the above matrix:

$$IDSDS = \{(b,x),(b,y),(c,x),(c,y)\}.$$

(b) Using your results from part (a), show that there is no pure strategy Nash equilibrium (psNE) in this game.

- Using the strategy profiles that survived IDSDS, we can next underline best response payoffs, as depicted in the matrix below.

| | | *Player 2* | |
|---|---|---|---|
| | | $x$ | $y$ |
| *Player 1* | $b$ | $\underline{5},1$ | $2,\underline{3}$ |
| | $c$ | $3,\underline{7}$ | $\underline{4},6$ |

Since there is no cell where both players' payoffs are underlined, we can claim that there is no pure strategy Nash equilibrium in this game. There is, however, a mixed strategy Nash equilibrium, as we show in the next part of the exercise!

(c) Using your results from part (a), find a mixed strategy Nash equilibrium (msNE) in this game.

- *Player 1.* If player 1 is randomizing, he must be indifferent between pure strategies $b$ and $c$. His expected utility from choosing $b$ (in the top row of the above matrix) is

$$EU_1(b) = 5q + 2(1-q) = 3q + 2$$

while his expected utility from selecting $c$ (in the bottom row of the matrix) is

$$EU_1(c) = 5q + 4(1-q) = 4 - q.$$

Then, player 1 is indifferent between $b$ and $c$ if and only if $EU_1(b) = EU_1(c)$, which implies that

$$3q + 2 = 4 - q$$

and, after rearranging, $4q = 2$, or $q = \frac{1}{2}$.

- *Player 2.* If player 2 is randomizing, he must be indifferent between his pure strategies $x$ and $y$. His expected utility from choosing $x$ (in the left-hand column of the above matrix) is

$$EU_2(x) = 1p + 7(1-p) = 7 - 6p$$

while his expected utility from selecting $y$ (in the right-hand column of the matrix) is

$$EU_2(y) = 3p + 6(1-p) = 6 - 3p.$$

Then, player 2 is indifferent between $x$ and $y$ if and only if $EU_2(x) = EU_2(y)$, which implies that

$$7 - 6p = 6 - 3p$$

or, after rearranging, $1 = 3p$, or $p = \frac{1}{3}$.

- Therefore, the mixed strategy Nash equilibrium of the game is

$$\left\{ \left( \frac{1}{3}b, \frac{2}{3}c \right), \left( \frac{1}{2}x, \frac{1}{2}y \right) \right\}$$

where the first pair indicates player 1's randomization between $b$ and $c$ with probabilities 1/3 and 2/3 respectively, while the second pair represents player 2's randomization between $x$ and $y$, each with 50% probability.