

EconS301 - Quiz #5

In Tacoma, WA there are two suppliers of distilled water, labeled as firm Aqua and firm Blue. Distilled water is considered a homogenous good. Let p denote the price per gallon, q_A quantity sold by firm Aqua, and q_B the quantity sold by firm Blue. Both firms are located relatively close to a spring, so the only production cost is the cost of bottling. Formally, each firm bears a production cost of $c_A = \$3$ and $c_B = \$2$ per one gallon of water.

The aggregate inverse demand function for distilled water is given by $p = 12 - 0.5Q$, where $Q = q_A + q_B$ denotes the aggregate industry supply of distilled water in Tacoma.

- a) Identify firm A's best-response function, $q_A = R_A(q_B)$. Also find firm B's best-response function, $q_B = R_B(q_A)$. Show your derivations.
- b) Solve for the Cournot equilibrium output levels q_A^c and q_B^c . State which firm sells more water (if any) and why.
- c) Identify the aggregate industry supply and the equilibrium price of distilled water in Tacoma.
- d) Identify the profit level made by each firm, and for the aggregate industry profit. Which firm earns a higher profit and why?

Solution:

- a) Each firm chooses its output to maximize profits taking its rival's output as given. Firm A solves

$$\pi_A = (12 - 0.5(q_A + q_B) - 3) q_A = (9 - 0.5q_A - 0.5q_B) q_A.$$

Taking the first-order condition with respect to q_A yields $9 - q_A - 0.5q_B = 0$, so firm A's best-response function is $q_A(q_B) = (18 - q_B)/2$, or equivalently $q_A(q_B) = 9 - 0.5 q_B$. Similarly, firm B solves

$$\pi_B = (12 - 0.5(q_A + q_B) - 2) q_B = (10 - 0.5q_A - 0.5q_B) q_B.$$

The first-order condition with respect to q_B gives $10 - 0.5q_A - q_B = 0$, so firm B's best-response function is $q_B(q_A) = (20 - q_A)/2$, or equivalently $q_B(q_A) = 10 - 0.5 q_A$.

- b) The above best-response functions constitute two linear equations with two variables, q_A and q_B . Substituting firm B's best response into firm A's best response yields $q_A = 9 - 0.5(10 - 0.5 q_A) = 4 + 0.25 q_A$, so $0.75 q_A = 4$ and thus $q_A^c = 16/3 \approx 5.33$. Plugging back into firm B's best response gives $q_B^c = 10 - 0.5(16/3) = 22/3 \approx 7.33$. Firm B sells more water ($22/3 > 16/3$) because it is more efficient, bearing a lower marginal production cost ($\$2 < \3).
- c) $Q = q_A + q_B = 16/3 + 22/3 = 38/3 \approx 12.67$ gallons. The equilibrium price is $p = 12 - 0.5(38/3) = 17/3 \approx \5.67 per gallon.
- d) Since there are no fixed costs, $\pi_A = (p - c_A)q_A = (17/3 - 3)(16/3) = (8/3)(16/3) = 128/9 \approx \14.22 . Similarly, $\pi_B = (p - c_B)q_B = (17/3 - 2)(22/3) = (11/3)(22/3) = 242/9 \approx \26.89 . Industry profit is then $\Pi = \pi_A + \pi_B = 128/9 + 242/9 = 370/9 \approx \41.11 . Firm B earns a higher profit because it is more efficient: its lower marginal cost allows it to both produce a larger quantity and enjoy a wider margin ($p - c_B > p - c_A$) on every gallon sold.