EconS 301- Intermediate Microeconomic Theory
Quiz #3 - March 12th, 2026

1. Consider a firm with production function $q = 5L^{1/3}K^{2/3}$ where L denotes units of labor and K represents units of capital. Assume that the firm seeks to produce $q = 220$ units of output, it faces input prices of $w = \$15$ per unit of labor capital, and $k = \$20$ per unit of capital.

   (a) Solve the firm's cost-minimization problem, to obtain the combination of inputs (labor and capital) that minimizes the firm's cost of producing an amount of output $q = 220$.

   - The tangency condition $\frac{MP_L}{MP_K} = \frac{w}{r}$ is

$$\frac{\frac{5}{3}L^{-2/3}K^{2/3}}{\frac{10}{3}L^{1/3}K^{-1/3}} = \frac{15}{20}$$

which simplifies to

$$\frac{1}{2}\frac{K}{L} = \frac{3}{4}$$

This contains both L and K, so we solve for K:

$$K = \frac{3}{4}2L = \frac{3}{2}L$$

We can insert this back into the firm's output target $220 = 5L^{1/3}K^{2/3}$, to find that

$$220 = q = 5L^{1/3}(\frac{3}{2}L)^{2/3}$$

rearranging,

$$220 = 5\left(\frac{3}{2}\right)^{2/3} L^{1/3+2/3},$$

where we need to solve for L:

$$L = 44\left(\frac{2}{3}\right)^{2/3}$$

$$L = 33.57$$

This is the firm's labor demand. Plugging this back into the tangency condition,

$$K = \frac{3}{2}L = \frac{3}{2} \times 33.57 \simeq 50.37$$

which is the firm's demand for capital.

   (b) Use your results from part (a) to find the firm's cost function. This is its long-run total cost, as all inputs can be altered.

- Plugging back into the cost function, we get

$$C = wL + rK = 15 \times \underbrace{33.57}_{L} + 20 \times \underbrace{50.37}_{K}.$$

If we simplify this numerically, we get

$$C \simeq 1510.90.$$

(c) Find the firm's marginal product and average product for labor. Interpret.

- The firm's marginal production is

$$MP_L = \frac{\partial q}{\partial L} = \frac{5}{3}\left(\frac{K}{L}\right)^{\frac{2}{3}}.$$

The firm's average production is

$$AP_L = \frac{q}{L} = \frac{5L^{1/3}K^{2/3}}{L} = \frac{5K^{2/3}}{L^{2/3}} = 5\left(\frac{K}{L}\right)^{\frac{2}{3}}.$$

Hence, the AP_L lies above the MP_L since $5\left(\frac{K}{L}\right)^{\frac{2}{3}} > \frac{5}{3}\left(\frac{K}{L}\right)^{\frac{2}{3}}$.