EconS 301- Intermediate Microeconomic Theory
Homework #4 - Due date: Thursday March 26th, 2026.

1. Sarah is looking into producing her homemade dog treats on a larger scale and is contemplating two different kitchen sizes (K). Her production of dog treats follows $q = 200KL + K^2L^3$. What are the marginal and average product curves for labor when $K = 5$? What happens to the marginal and average product curves when her kitchen doubles to $K = 10$?

2. Is it possible for two firms with different Cobb-Douglas production functions, $q_1 = AL^\alpha K^\beta$ and $q_2 = L^\alpha K^\beta$, to have the same marginal products at particular levels of capital and labor? What about each firms' marginal rate of technical substitution.

3. A firm has the following cost function:

$$TC(q) = 2q^3 - \frac{1}{3}q^2 + \frac{1}{2}q + \frac{9}{10},$$

   where q denotes units of output. Intuitively, the first three terms on the right side capture the firm's variable cost, because they depend on the output the firm produces, whereas the last term represents its fixed cost, as it is not a function of output q.

   (a) *Total cost.* For which output q does the total cost curve $TC(q)$ increase or decrease? For which values is it concave or convex in output?

   (b) *Marginal cost.* For which output q does the marginal cost curve $\frac{\partial TC(q)}{\partial q}$ increase or decrease? For which values is it concave or convex in output?

   (c) *Average cost.* For which output q does the average cost curve $AC(q) = \frac{TC(q)}{q}$ increase or decrease? For which values is it concave or convex in output?

   (d) *Average variable cost.* For which output q does the average variable cost curve $AC(q)$ increase or decrease? For which values is it concave or convex in output?

   (e) Find the value of q where the marginal cost curve crosses the total cost curve, where it crosses the average cost curve, and where it crosses the average variable cost curve.

4. Consider a firm with the Cobb-Douglas production function $f(K, L) = 4K^{1/2}L^{1/3}$, where K denotes units of capital and L represents units of labor. Assume that the firm faces input prices of $r = \$10$ per unit of capital, and $w = \$7$ per unit of labor.

   (a) Solve the firm's cost-minimization problem, to obtain the combination of inputs (labor and capital) that minimizes the firm's cost of producing a given amount of output, q.

   (b) Use your results from part (a) to find the firm's cost function. This is its long-run total cost, as all inputs can be altered.

   (c) Find the firm's marginal cost function, and its average cost function. Interpret.

   (d) Assume now that the amount of capital is held fixed at $\overline{K} = 3$ units. Solve the firm's cost-minimization problem again to find the amount of labor that minimizes the firm's cost.

(e) Use your results from part (d) to find the firm's short-run cost function (since in the short run the firm can only alter the amount of labor, but without changing the units of capital).