EconS 301- Intermediate Microeconomic Theory
Homework #3 - Due date: Thursday March 5th, 2026.

1. Every spring, John goes to a local co-op to buy seeds to plant in his field. He has been keeping track of prices of seeds and the number of tons of seeds that he buys each year and estimates his demand curve to be $p(q) = 300 - 10q$. Last year, John paid \$150 per ton of seeds. This year, he noticed that the price went down to \$100. Unfortunately, John didn't take any economics courses in college, so he doesn't know how to quantify his welfare improvement. Help John find his CS from this price decrease.

   - Last year, John paid \$150 per ton of seed and purchased the amount as given by his demand curve
   $$\$150 = 300 - 10q,$$
   or $q = 15$ units, with corresponding consumer surplus
   $$CS = \frac{1}{2}(300 - 150)15 = 1,125.$$
   - When the price decreases to \$100, he buys according to
   $$\$100 = 300 - 10q,$$
   or $q = 20$ units, with corresponding consumer surplus
   $$CS' = \frac{1}{2}(300 - 100)20 = 2,000.$$
   Therefore, John's consumer surplus increases by
   $$CS' - CS = 2000 - 1125 = 875.$$

2. Chris has a demand for books (b) and other goods (y) that follows the Cobb-Douglas utility function $u(b, y) = y\sqrt{b}$, and an income of $I = \$50$. Find Chris's CV if the price of books decreases from $p_b = \$2$ to $p'_b = \$1$.

   - For this demand, the tangency condition $\frac{MU_b}{MU_y} = \frac{p_b}{p_y}$, or $\frac{y}{2\sqrt{b}\sqrt{b}} = \frac{p_b}{p_y}$, which simplifies to $y = 2p_b b$ since $p_y = \$1$. Substituting this into the budget line $p_b b + p_y y = I$ we obtain $p_b b + 2p_b b = 50$, which simplifies to $3p_b b = 50$, and demand for books b is $b = \frac{50}{3p_b}$. Demand for other good y is
   $$y = 2p_b\frac{50}{3p_b} = \frac{100}{3} = 33.33.$$

     (a) *Finding initial bundle A.* At the initial price of $p_b = 2$, the demand for books is $b_A = \frac{50}{3\times2} = 8.33$ books.
     (b) *Finding final bundle C.* At the final price of $p'_b = 1$, the demand for books is $y_A = \frac{50}{3\times1} = 16.66$ books.
     (c) *Finding the decomposition bundle B.* At the decomposition bundle, the consumer must:

1. Reach the same utility level as with the initial bundle $A = (8.33, 33.33)$. This bundle yields a utility level of

$$u_A = 33.33\sqrt{8.33} = 96.20.$$

The decomposition bundle must also yield a utility level of 96.20, which we can write as

$$(y_B)\sqrt{b_B} = 96.20.$$

2. The consumer's indifference curve must be tangent to the budget line, $y = 2p'_b b$, or $y = 2b$ since $p'_b = \$1$. Substituting this into the equation above,

$$(2b_B)\sqrt{b_B} = 96.20.$$

Simplifying, we obtain $b_B^{3/2} = \frac{96.20}{2} = 48.1$. Solving for b_B, we get

$$b_B = 40.1^{2/3} \simeq 13.22 \text{ units.}$$

We can insert this into the tangency condition to find that

$$y_B = 2b_B = 2 \times 13.22 = 26.44 \text{ units.}$$

The decomposition bundle is, then, $B = (13.22, 26.44)$.

3. *Evaluating the CV.* The CV is given by $CV = I - I_B$, where $I = \$50$ and I_B is

$$I_B = 13.22 + 26.44 = 39.66.$$

The CV is

$$CV = I - I_B = 50 - 39.66 = 10.34.$$

After the price decrease, we decrease Chris's income by \$10.34, and his utility level coincides with that before the price decrease.

3. You are looking at two firms as an investment opportunity.

- For the first firm, you know that with probability 0.7, your investment will mature to a profit of \$45 million, and with probability 0.3, your investment will mature to a loss of \$30 million.
- For the second firm, you know that with probability 0.8, your investment will mature to a profit of \$30 million, and with probability 0.2, your investment will mature to a loss of \$7.5 million.

(a) Calculate the EV of each investment.

- *First firm.* The expected value of investing in the first firm is

$$\begin{aligned} EV_1 &= (0.7 \times \$45) - (0.3 \times \$30) \\ &= 31.5 - 9 \\ &= \$22.5 \text{ million.} \end{aligned}$$

- *Second firm.* The expected value of investing in the second firm is

$$\begin{aligned} EV_2 &= (0.8 \times \$30) - (0.2 \times \$7.5) \\ &= 24 - 1.5 \\ &= \$22.5 \text{ million.} \end{aligned}$$

(b) Calculate the variance of each investment.

- *First firm.* The variance of investing in the first firm is

$$\begin{aligned} Var_1 &= 0.7(\$45 - \$22.5)^2 + 0.3(-\$30 - \$22.5)^2 \\ &= 0.7(\$22.5)^2 + 0.3(-\$52.5)^2 \\ &= 0.7(506.25) + 0.3(2756.25) \\ &= \$1,181.25 \text{ million.} \end{aligned}$$

- *Second firm.* The variance of investing in the second firm is

$$\begin{aligned} Var_2 &= 0.8(\$30 - \$22.5)^2 + 0.2(-\$7.5 - \$22.5)^2 \\ &= 0.8(\$7.5)^2 + 0.2(-\$30)^2 \\ &= 0.8(56.25) + 0.2(900) \\ &= \$225 \text{ million.} \end{aligned}$$

(c) If you had the opportunity to invest in only one of these firms, which would you pick and why?

- Each investment has the same expect value; however, the investment in the second firm offers the lower variance. The risk neutral investor would likely invest in the second firm due to the lower variance, but a more risky individual may choose to invest in the first firm.

4. Suppose that you took part in a lottery that had a chance to increase, decrease, or have no effect on your level of income. With probability 0.5, your income remains at its original level, \$500. With probability 0.2, your income increases to \$700, and with probability 0.3, your income decreases to \$400. Your utility function is

$$u(I) = I^{0.7},$$

where I denotes your income level.

(a) Using only the utility function, show that your risk preferences are risk averse.

- We know that utility functions following the form $u(I) = a + bI^\gamma$ are concave where a and b are positive, and $\gamma \in (0, 1)$. For our utility function, $a = 0$, $b = 1$, and $\gamma = 0.7$. This fits the requirements for a concave utility function, which means that we are risk averse.

(b) Calculate both your EU and the utility equivalent of the EV of your income.

- *Expected utility.* The expected utility of our income is

$$\begin{aligned} EU &= 0.5(500^{0.7}) + 0.2(700^{0.7}) + 0.3(400^{0.7}) \\ &= 0.5(77.50) + 0.2(98.08) + 0.3(66.29) \\ &= 78.25. \end{aligned}$$

- *Utility of expected value.* First, we need to find our expected income; that is,

$$\begin{aligned} EV &= (0.5 \times \$500) + (0.2 \times \$700) + (0.3 \times \$400) \\ &= \$250 + \$140 + \$120 \\ &= \$510. \end{aligned}$$

  Our utility at the expected value is

$$u(510) = 510^{0.7} = 78.58.$$

(c) Using the results from part (b), show that your risk preferences are risk averse.

- To show that we are risk averse, we need that $u(EV) > EU$, which holds in this case since

$$78.58 = u(EV) > EU = 78.25.$$

(d) Suppose now that you had the option to either accept this lottery, or walk away with your initial \$500. Should you accept the lottery? Why or why not?

- With our initial income, our utility is

$$u(500) = 500^{0.7} = 77.50.$$

  Since this is less than our expected utility from accepting the lottery ($EU = 78.25$), we should accept the lottery.

(e) Calculate your certainty equivalent.

- The certainty equivalent is found by solving

$$u(CE) = EU,$$

  where $EU = 78.25$ as found in Exercise 6.7, and $u(I) = I^{0.7}$. The previous equation becomes

$$CE^{0.7} = 78.25$$

  Taking each side to the 1/0.7 power, we find the certainty equivalent

$$CE = 78.25^{1/0.7} = 506.97.$$

(f) Calculate and interpret your risk premium. Is it consistent with risk aversion?

- The risk premium RP is found by solving the following

$$CE = EV - RP.$$

Plugging in $EV = 510$ and $CE = 504.65$, we obtain

$$506.97 = 510 - RP,$$

and solving for RP gives us our risk premium

$$RP = \$3.03.$$

Therefore, we would need to decrease the expected value of the lottery by \$3.03 to feel indifferent on accepting the lottery and taking the certain payout of the expected value. Since this value is positive, and we need to receive money to prefer the lottery, we again confirm that we are risk averse.

5. Consider a situation where you are faced with a risky situation. You currently have \$100,000 available for consumption, and with a 90 percent probability, you would suffer no illness. You have a 9 percent chance, however, of contracting a case of influenza, leading to the loss of \$10,000 in consumption. In addition, there is a 1 percent chance that this is a severe illness, leading to the loss of \$50,000 in consumption. Your utility from consumption is

$$U(C) = C^{0.4},$$

where C is your consumption level.

(a) What is your attitude toward risk? How do you know this?

- We know that utility functions following the form $u(I) = a + bI^{\gamma}$ are concave if $a, b > 0$ and $\gamma \in (0, 1)$. For our utility function, $a = 0$, $b = 1$, and $\gamma = 0.4$. This fits the requirements for a concave utility function, which means that we are risk averse.

(b) Suppose that you could purchase insurance against influenza. What is your CE?

- Our certainty equivalent is

$$CE = EV - RP.$$

We first need to calculate expected value; that is,

$$EV = (0.9 \times \$100,000) + (0.09 \times (\$100,000 - \$10,000)) + (0.01 \times (\$100,000 - \$50,000))$$

which simplifies to

$$\begin{aligned} EV &= (0.9 \times \$100,000) + (0.09 \times \$90,000) + (0.01 \times \$50,000) \\ &= \$90,000 + \$8,100 + \$500 \\ &= \$98,600 \end{aligned}$$

Therefore, our certainty equivalent is

$$CE = 98,600 - RP.$$

(c) What is the maximum premium that you are willing to pay for insurance against influenza?

- The maximum we are willing to pay for insurance against influenza is the difference between our utility at our initial income less the insurance premium (IP) and our expected utility. We first need to find our expect utility, which is

$$\begin{aligned} EU &= 0.9(100,000^{0.4}) + 0.09(90,000^{0.4}) + 0.01(50,000^{0.4}) \\ &= 0.9(100) + 0.09(95.87) + 0.01(75.79) \\ &= 99.39. \end{aligned}$$

  The utility from purchasing insurance is

$$u(100,000 - IP) = (100,000 - IP)^{0.4}.$$

  Setting the above expression equal to our expected utility of 99.39, we find that

$$(100,000 - IP)^{0.4} = \underbrace{99.39}_{EU}.$$

  Taking each side to the 1/0.4 power, we get that

$$100,000 - IP = 98,482.$$

  Solving for IP, we find that the maximum we would pay for insurance is

$$IP = \$1,519.$$

(d) What is your risk premium? How does this compare with your risk premium if you were risk neutral?

- *Risk averse.* The maximum we are willing to pay for insurance against influenza is the risk premium RP, which we can find by solving $u(CE) = EU$, or

$$u(98,600 - RP) = 99.39.$$

  With our utility function, this equation becomes

$$(98,600 - RP)^{0.4} = 99.39.$$

  Taking each side to the $\frac{1}{0.4}$ power, we obtain

$$98,600 - RP = 98,482,$$

  and solving for RP, we find that

$$RP = \$118.$$

- *Risk neutral.* If we were risk neutral, with a utility of $u(I) = I$ (a linear utility function), our expected utility coincides with the expected value, i.e.,

$$\begin{aligned} EU &= (0.9 \times \$100,00) + (0.09 \times \$90,000) + (0.01 \times \$50,000) \\ &= \$90,000 + \$8,100 + \$500 \\ &= \$98,600. \end{aligned}$$

Therefore, to find our risk premium in this case, we solve

$$u(98,600 - RP) = 98,600.$$

Given that our utility function is $u(I) = I$, this equation becomes

$$98,600 - RP = 98,600,$$

and our risk premium is \$0. If we were risk neutral, we would have a lower risk premium than if we are risk averse.