EconS 301- Intermediate Microeconomic Theory Midterm #1 - February 12th, 2026.

1. Olivia's preferences for chocolate, x, and milk, y, can be represented with the following Cobb-Douglas utility function $u(x, y) = x^3y^2$.

   (a) Find Olivias's marginal utility for books, MU_x, and for computers, MU_y.

   - We calculate Olivias's marginal utilities by differentiating with respect to each respective variable,

   $$MU_x = \frac{\partial u(x,y)}{\partial x} = 3x^2y^2 \qquad MU_y = \frac{\partial u(x,y)}{\partial y} = 2x^3y$$

   (b) Are her preferences monotonic (i.e., weakly increasing in both goods)?

   - Since both of Olivia's marginal utilities are strictly positive, her preferences are monotonic.

   (c) For a given utility level, $\overline{u}$, solve the utility function for y to obtain Olivia's indifference curve.

   - Setting Olivia's utility at $\bar{u}$, we obtain

   $$x^3y^2 = \bar{u}$$

   Solving Eric's utility function for y, we find her indifference curve,

   $$y = \sqrt{\frac{\bar{u}}{x^3}}.$$

   (d) Find Olivia's marginal rate of substitution between x and y (MRS). Interpret your results.

   - Olivia's marginal rate of substitution can be found by taking the ratio of her marginal utility with respect to x to his marginal utility with respect to y,

   $$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{3x^2y^2}{2x^3y} = \frac{3y}{2x}$$

   Olivia's marginal rate of substitution implies that for each additional unit of books she receives, she must be compensated with an additional $\frac{3}{2}$ units of computers in order to give up any further units of books.

   (e) Are her preferences convex (i.e., bowed-in towards the origin)?

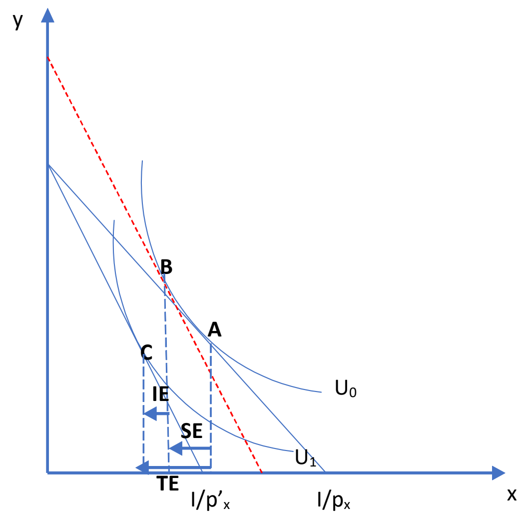
   - The convexity of Amelia's preferences can be found by differentiating his marginal rate of substitution with respect to x,

   $$\frac{dMRS_{x,y}}{dx} = -\frac{3y}{2x^2} < 0$$

   Since this is negative, Amelia's marginal rate of substitution is decreasing in x, which implies that Amelia's preferences are convex.

2. Assume that an individual consumes two goods x and y, where the price of good x is p_x, the price of good y is p_y and her income is denoted by I. Let us consider a shortage of good x which makes this good more expensive, the new price becomes p'_x where $p'_x > p_x$ and the price of good y is unaffected. Using a graph, identify and discuss the *substitution effect*, *income effect* and *total effect* of this increase in price when good x is a normal good.

   - Let us consider the following figure



   - We observe that an increase in price shifts the initial budget line inwards. The individual's utility decreases to U_1 when the price of good x increases. In addition, the individual consumes a new bundle, bundle C, which contains less units of good x. The movement from bundle A to bundle C represents the total effect of a decrease in price, which is negative. The substitution effect and income effect are also negative representing a decrease in the consumption of good x. The substitution effect is represented by a movement along the initial indifference curve (from bundle A to bundle B) and the income effect is the jump from bundle B to bundle C. Note that in this case the individual needs to receive wealth in order to maintain her initial utility U_o (represented by the dotted line, which has the same slope than the new budget line).

3. Amelia wishes to reach a utility level of $U = 50$ and has a quasilinear utility function of the type $u(x, y) = 4x + y^{1/2}$. The price of good x is \$2 while the price of good y is also \$2.

   (a) Find Amelia's tangency condition.

      - *Step 1.* We begin the expenditure minimization problem by finding Amelia's tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$.

- Calculating Amelia's marginal utilities, we obtain $MU_x = \frac{\partial u(x,y)}{\partial x} = 4$ and $MU_y = \frac{\partial u(x,y)}{\partial y} = \frac{1}{2}y^{-1/2}$. Combining these into our marginal rate of substitution gives

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{4}{\frac{1}{2}y^{-1/2}} = 8y^{1/2}.$$

- Next, we obtain our ratio of prices, $\frac{p_x}{p_y} = \frac{2}{2}$.
- Setting them equal to one another, we have our tangency condition,

$$8y^{1/2} = 1.$$

(b) Find Amelia's equilibrium quantities for goods x and y.

- Since Amelia's tangency condition contains only y we move on to step 2b.
- *Step 2b.* Rearranging this expression, we have $y^{1/2} = \frac{1}{8}$. Squaring both sides gives our optimal purchase of good y,

$$y^E = \frac{1}{64} = 0.015 \text{ units.}$$

  - Now we substitute this value, $y^E = 0.015$, into our utility function, $4x + y^{1/2} = 50$ to obtain

$$4x + (0.125) = 50,$$

  or $4x + 0.125 = 50$, which further simplifies to $4x = 49.5$. We can now solve for x, to obtain the optimal purchase of good x,

$$x^E = 12.47 \text{ units.}$$

(c) How much income does Amelia require to reach her target utility level?

- To find Amelia's income requirement, we multiply her equilibrium quantities by their respective prices and add them together, obtaining

$$I = 2(12.375) + 2(0.125) = \$25$$

4. Chelsea's utility function is $u(x, y) = 3x^{1/2} + 4y$, her income is $I = \$220$, and $p_y = \$1$. The price of good x decreases from $p_x = \$3$ to $p_x' = \$2$. Using the steps in Example 4.9, find the substitution and income effect.

- To begin our analysis, it is useful to consider Chelsea's marginal rate of substitution,

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{\frac{3}{2}x^{-1/2}}{4} = \frac{3}{8x^{1/2}}.$$

- *Finding initial bundle* A. Using Chelsea's initial prices, we can establish her tangency condition, $\frac{MU_x}{MU_y} = \frac{3}{1}$, as

$$\frac{3}{8x^{1/2}} = 3.$$

Solving this expression for x, we obtain

$$x = \left(\frac{1}{8}\right)^2 = \frac{1}{64} = 0.0156 \text{ units.}$$

Using Chelsea's budget line, we can substitute this result in to obtain $3\left(\frac{1}{64}\right) + y = 220$. Solving this expression for x, we obtain Chelsea's initial consumption of good y,

$$y = 220 - \frac{3}{64} = 219.95 \text{ units.}$$

This gives us Chelsea's initial bundle $A = (0.0156, 219.95)$.

- *Finding final bundle* C. Now with Chelsea's final prices, her tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$ is

$$\frac{3}{8x^{1/2}} = \frac{2}{1}.$$

Once again, we can solve this expression for x to obtain

$$x = \left(\frac{3}{16}\right)^2 = \frac{9}{256} = 0.0352 \text{ units.}$$

As before, we use Chelsea's budget line and substitute this result in to obtain $2\left(\frac{9}{256}\right) + y = 220$. Solving this expression for y, we obtain Chelsea's final consumption of good y,

$$y = 220 - \frac{18}{256} = 219.93 \text{ units,}$$

which provides Chelsea's final bundle $C = (0.0352, 219.93)$.

- *Finding decomposition bundle* B. Now we need to determine how Chelsea's bundle changes from her initial bundle to his final bundle, breaking them into the substitution and income effects. To calculate this, we need to satisfy both conditions as explained in example 4.9.
  - First, the decomposition bundle must reach the utility level Chelsea received under her initial bundle, which we can calculate by inserting bundle A into Chelsea's utility function,

    $$3(0.0156)^{1/2} + 4(219.95) = 880.18.$$

    Thus, Chelsea's decomposition bundle B must satisfy $3x^{1/2} + 4y = 880.18$.
  - Second, the decomposition bundle must be tangent to Chelsea's indifference curve. This happens where the slope of Chelsea's indifference curve is equal to the slope of her final budget line, which implies that we use the tangency condition from Chelsea's final bundle, $x = 0.0352$.

Substituting Chelsea's tangency condition into the first condition gives

$$3\left(\frac{9}{256}\right)^{1/2} + 4y = 880.18.$$

Since $3\left(\frac{9}{256}\right)^{1/2} = 0.5625$, we can solve this expression for good y to obtain,

$$x = \frac{880.18 - 0.5625}{4} = 219.90 \text{ units.}$$

This gives us Chelsea's decomposition bundle $B = (0.0352, 219.90)$.

- Lastly, we can calculate Chelsea's substitution and income effects by comparing the values of her different bundles:

$$\begin{aligned} \text{Substitution Effect:} &\quad x_B - x_A = 0.0352 - 0.0156 = 0.0196 \\ \text{Income Effect:} &\quad x_C - x_B = 0.0352 - 0.0352 = 0 \end{aligned}$$

  This result for Chelsea should make sense. Since the majority of her utility comes from consuming good y (the quasilinear parameter), Chelsea allocates all of her savings from the price change to purchasing more of good y. This leads to zero income effect with all of the gains due to Chelsea substituting from good y to good x.

5. Consider an individual with utility function $u(x, y) = \min\{2x, 3y\}$, facing an income $I = 250$ and prices p_x and p_y for goods 1 and 2, respectively.

   (a) Find the demand function for each good.

   - We must first determine the preferred consumption ratio given that we have perfect complements. Since we receive our utility as the lesser of $2x$ and $3y$, he prefers to consume them in that ratio, $2x = 3y$, or $x = \frac{3}{2}y$. What we derived is essentially a tangency condition. Since our tangency condition contains both x and y we move on to step 2a from the utility maximization procedure in chapter 3.
   - *Step 2a.* Next, we use our budget line, $p_x x + p_y y = I$ and substitute $\frac{3}{2}y$ for x, obtaining

     $$p_x \underbrace{\left(\frac{3}{2}y\right)}_{x} + p_y y = I.$$

     - Combining terms, we have $\left(\frac{3}{2}p_x + p_y\right) y = I$, and we can divide both sides of this equation to obtain the amount of good y that he consumes,

       $$y = \frac{I}{\frac{3}{2}p_x + p_y} = \frac{2I}{3p_x + 2p_y} \text{ units.}$$

       With a positive value of y, we can move on to step 4.
   - *Step 4.* Lastly, we return to our tangency condition to determine how much of good x he consumes.
     - Plugging in our value for y gives

       $$x = \frac{3}{2}\left(\frac{2I}{3p_x + 2p_y}\right) = \frac{3I}{3p_x + 2p_y} \text{ units.}$$

- Lastly, plugging in our value for income of $I = \$250$, we have our final demand functions,

$$x = \frac{750}{3p_x + 2p_y} \qquad y = \frac{500}{3p_x + 2p_y}$$

(b) Calculate the price elasticity of demand and income elasticities for both goods. Interpret.

- *Price Elasticity of Demand:*
  - *Good x.* Using the formula for the price elasticity of demand,

    $$\varepsilon_{x,p_x} = \frac{\partial x}{\partial p_x}\frac{p_x}{x}.$$

    For the first term, we can differentiate the demand function with respect to p_x to obtain,

    $$\frac{\partial x}{\partial p_x} = -\frac{9I}{(3p_x + 2p_y)^2}.$$

    In addition, we can substitute $\frac{3I}{3p_x+2p_y}$ for x in the price elasticity of demand formula, obtaining

    $$\begin{aligned} \varepsilon_{x,p_x} &= \frac{\partial x}{\partial p_x}\frac{p_x}{x} = -\frac{9I}{(3p_x + 2p_y)^2}\frac{p_x}{\frac{3I}{3p_x+2p_y}} \\ &= -\frac{9Ip_x(3p_x + 2p_y)}{3I(3p_x + 2p_y)^2} = -\frac{3p_x}{3p_x + 2p_y}. \end{aligned}$$

    The price elasticity of demand in this case is negative (as expected), but greater than -1, implying that the price elasticity of demand is inelastic. Thus, the consumer is not too sensitive to the price of good x.
  - *Good y.* Using the formula for the price elasticity of demand,

    $$\varepsilon_{y,p_y} = \frac{\partial y}{\partial p_y}\frac{p_y}{y}.$$

    For the first term, we can differentiate the demand function with respect to p_y to obtain,

    $$\frac{\partial y}{\partial p_y} = -\frac{4I}{(3p_x + 2p_y)^2}.$$

    In addition, we can substitute $\frac{2I}{3p_x+2p_y}$ for y in the price elasticity of demand formula, obtaining

    $$\varepsilon_{y,p_y} = \frac{\partial y}{\partial p_y}\frac{p_y}{y} = -\frac{4I}{(3p_x + 2p_y)^2}\frac{p_y}{\frac{2I}{3p_x+2p_y}} = -\frac{4Ip_y(3p_x + 2p_y)}{2I(3p_x + 2p_y)^2} = -\frac{2p_y}{3p_x + 2p_y}.$$

    The price elasticity of demand in this case is negative (as expected), but greater than -1, implying that the price elasticity of demand is inelastic. Thus, the consumer is not too sensitive to the price of good y.
- *Income Elasticity:*

- *Good x.* Using the formula for the income elasticity,

$$\varepsilon_{x,I} = \frac{\partial x}{\partial I}\frac{I}{x}.$$

For the first term, we can differentiate the demand function with respect to I to obtain,

$$\frac{\partial x}{\partial I} = \frac{3}{3p_x + 2p_y}.$$

In addition, we can substitute $\frac{3I}{3p_x+2p_y}$ for x in the price elasticity of demand formula, obtaining

$$\varepsilon_{x,I} = \frac{\partial x}{\partial I}\frac{I}{x} = \frac{3}{3p_x + 2p_y}\frac{I}{\frac{3I}{3p_x+2p_y}} = \frac{3I(3p_x + 2p_y)}{3I(3p_x + 2p_y)} = 1.$$

With an income elasticity equal to 1, we know that this is a normal good, and that a 1% increase in income will correspond with a 1% increase in the consumption of good x, as we would expect with perfect complements.

- *Good y.* Using the formula for the income elasticity,

$$\varepsilon_{y,I} = \frac{\partial y}{\partial I}\frac{I}{y}.$$

For the first term, we can differentiate the demand function with respect to I to obtain,

$$\frac{\partial y}{\partial I} = \frac{2}{3p_x + 2p_y}.$$

In addition, we can substitute $\frac{2I}{3p_x+2p_y}$ for y in the price elasticity of demand formula, obtaining

$$\varepsilon_{y,I} = \frac{\partial y}{\partial I}\frac{I}{y} = \frac{2}{3p_x + 2p_y}\frac{I}{\frac{2I}{3p_x+2p_y}} = \frac{2(3p_x + 2p_y)}{2(3p_x + 2p_y)} = 1.$$

With an income elasticity equal to 1, we know that this is a normal good, and that a 1% increase in income will correspond with a 1% increase in the consumption of good y, as we would expect with perfect complements.