EconS 301- Intermediate Microeconomic Theory
Homework #2 - Due date: Tuesday February 10th, 2026.

1. Fran has utility function $u(x,y) = x^2y^{1/2}$, facing prices $p_x = \$5$ and $p_y = \$3$, and income $I = \$30$. Using the same steps as in example 3.2 (Chapter 3, page 52), find Fran's optimal consumption of goods x and y.

   - *Step 1.* To solve for Fran's optimal consumption, first we must use the tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$.
     - For the left-hand side, we calculate the ratio of our marginal utilities, where $MU_x = \frac{\partial u(x,y)}{\partial x} = 2xy^{1/2}$ and $MU_y = \frac{\partial u(x,y)}{\partial y} = \frac{1}{2}x^2y^{-1/2}$. Therefore,
     $$\frac{MU_x}{MU_y} = \frac{2xy^{1/2}}{\frac{1}{2}x^2y^{-1/2}} = \frac{4y}{x}.$$
     - For the right-hand side, it is simply the ratio of the prices, $\frac{p_x}{p_y} = \frac{5}{3}$.
     - Setting them equal to one another gives
     $$\frac{4y}{x} = \frac{5}{3},$$
     which we can rearrange to obtain $12y = 5x$. Since this contains both x and y we move on to step 2a.
   - *Step 2a.* Next, we use Fran's budget line, $5x + 3y = 30$ and substitute $12y$ for $5x$, obtaining
   $$\underbrace{12y}_{5x} + 3y = 30.$$
     - Combining terms, we have $15y = 30$, and we can divide both sides of this equation to obtain the amount of good y that Fran consumes, $y = 2$ units. With a positive value of y, we can move on to step 4.
   - *Step 4.* Last, we return to our tangency condition to determine how much of good x Fran consumes.
     - Dividing both sides of the tangency condition by 5, we have $x = \frac{12}{5}y$. Plugging in our value for y gives
     $$x = \frac{12}{5}(2) = 4.8 \text{ units}.$$

2. John's utility function is $u(x,y) = 6x + 8y$ and faces prices $p_x = \$2$ and $p_y = \$3$ and income $I = \$25$. Comparing his $MRS_{x,y}$ and the price ratio, find his optimal consumption of goods x and y.

   - First, we need to calculate John's marginal rate of substitution,
   $$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{6}{8} = 0.75,$$
   and compare it to the ratio of prices,
   $$\frac{p_x}{p_y} = \frac{2}{3} = 0.667$$

- Since $\frac{6}{8} > \frac{2}{3}$, John receives more benefit by solely consuming good x. Alternatively, the "bang for the buck" he obtains from good x, $\frac{MU_x}{p_x} = \frac{6}{2} = 3$, is larger than that for good y, $\frac{MU_y}{p_y} = \frac{8}{3} = 2.667$, inducing him to keep increasing his purchases of good x, while reducing those of good y, until he only consumes the former.
- In this case, John can consume

$$x = \frac{I}{p_x} = \frac{25}{2} = 12.5 \text{ units}$$

of good x and no units of good y.

3. Felix's utility function is $u(x, y) = 5\min\{4x, 6y\}$ and he faces prices $p_x = \$1$ and $p_y = \$2$ and income $I = \$120$. Find his optimal consumption of goods x and y.

- Since we cannot define John's marginal rate of substitution, we must use the fact that John prefers to consume goods x and y in fixed proportions to maximize his utility (i.e., the two arguments inside the min operator must coincide, $4x = 6y$) or, after dividing both sides by 4,

$$x = \frac{6}{4}y = \frac{3}{2}y.$$

This is the kink of Felix's indifference curves, as depicted in the following figure.

- Using the condition we found above, $x = \frac{3}{2}y$, and Felix's budget line, $x+2y = 120$, we can substitute $\frac{3}{2}y$ for x, giving the equation

$$\underbrace{\frac{3}{2}y}_{x} + 2y = 120.$$

  - Combining terms on the left-hand side of the equation gives us $\frac{7}{2}y = 120$. Dividing both sides of this expression by $\frac{7}{2}$ provides our equilibrium consumption level of good y, $y = 34.29$ units.

- Returning to our condition for Felix's consumption bundle, $x = \frac{3}{2}y$, we can plug in our value of $y = 34.29$ to find our equilibrium consumption level of good x,

$$x = \frac{3}{2}\underbrace{(34.29)}_{y} = 51.43 \text{ units.}$$

4. Brandon has a weekly income of $I = \$40$ that he allocates between purchasing goods x and y. When the price of good x is \$4 and the price of good y is \$4, Brandon purchases 3 units of good x and 7 units of good y in equilibrium. Suppose now that the price of good x falls to \$2.

   (a) Find the equation of his original and new budget lines, and represent it graphically.

- Setting up Brandon's original budget line, we have

$$4x + 4y = 40,$$

which, solving for y, we obtain,

$$y = 10 - x.$$

From here, we find the vertical intercept of Brandon's budget line by setting $x = 0$, which yields $y = 10$ units. Next, we find the horizontal intercept of Eric's budget line by setting $y = 0$ and solving for x,

$$0 = 10 - x$$

which yields $x = 10$ units.

- Setting up Brandon's new budget line, we have

$$2x + 4y = 40,$$

which, solving for y, we obtain,

$$y = 10 - \frac{1}{2}x.$$

From here, we find the vertical intercept of Brandon's budget line by setting $x = 0$, which yields $y = 10$ units. Next, we find the horizontal intercept of Brandon's budget line by setting $y = 0$ and solving for x,

$$0 = 10 - \frac{1}{2}x$$

which yields $x = 20$ units. Intuitively, this implies that Brandon's budget line rotates outward on the x-axis as a result of the price decrease. The resulting budget lines are plotted in figure 3.17.

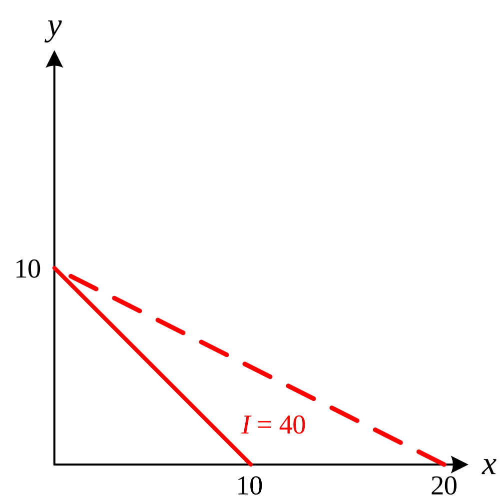

Figure 3.17. Budget lines.

(b) Suppose that Brandon's new equilibrium bundle is 5 units of good x and 5 units of good y. Does this new bundle violate WARP? Explain why or why not.

- Using Brandon's original prices, this bundle had a total cost of $4(5) + 4(5) =$ \$40. This implies that Brandon's new bundle was affordable under his original prices. In addition, Brandon's original bundle is also affordable under the new prices, as it now only costs $2(3) + 4(7) =$ \$34 to purchase it under the new prices. This implies that Brandon's new bundle violates WARP as both of his bundles were affordable under the original prices, while the original bundle is still affordable under the new prices.

(c) Suppose now that Brandon's new equilibrium bundle contained 4 units of good y. How many units of good x must be consumed such that our equilibrium allocation does not violate WARP?

- In order for this new bundle to not violate WARP, it must be unaffordable under the original prices, which would mean that the premise of WARP does not hold and thus cannot be violated. To set this up, we must have that,

$$4x + 4(4) > 40.$$

  Solving this expression for x, we have $x > 6$.
- Second, this bundle must be affordable under the new prices. To set this up, we must have that

$$2x + 4(4) \leq 40$$

  Solving this expression for x, we have $x \leq 12$. Thus, any bundle satisfying $6 < x \leq 12$. would not violate WARP in this context.

5. Carter wishes to reach a utility level of $U = 70$ and has a quasilinear utility function of the type $u(x, y) = 2x + y^{1/2}$. The price of good x is \$4 while the price of good y is \$1.

(a) Find Carter's tangency condition following step 1 of the expenditure minimization procedure.

- *Step 1.* We begin the expenditure minimization problem by finding Carter's tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$.
  - Calculating Carter's marginal utilities, we obtain $MU_x = \frac{\partial u(x,y)}{\partial x} = 2$ and $MU_y = \frac{\partial u(x,y)}{\partial y} = \frac{1}{2}y^{-1/2}$. Combining these into our marginal rate of substitution gives

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{2}{\frac{1}{2}y^{-1/2}} = 4y^{1/2}.$$

  - Next, we obtain our ratio of prices, $\frac{p_x}{p_y} = \frac{4}{1}$.
  - Setting them equal to one another, we have our tangency condition,

$$4y^{1/2} = 4.$$

(b) Find Carter's equilibrium quantities for goods x and y.

- Since Carter's tangency condition contains only y we move on to step 2b.
- *Step 2b.* Rearranging this expression, we have $y^{1/2} = 1$. Squaring both sides gives our optimal purchase of good y,

$$y^E = 1 \text{ units.}$$

  – Now we substitute this value, $y^E = 1$, into our utility function, $2x + y^{1/2} = 70$ to obtain

$$2x + (1)^{1/2} = 70,$$

  or $2x + 1 = 70$, which further simplifies to $2x = 69$. We can now solve for x, to obtain the optimal purchase of good x,

$$x^E = 34.5 \text{ units.}$$

(c) How much income does Carter require to reach his target utility level?

- To find Carter's income requirement, we multiply his equilibrium quantities by their respective prices and add them together, obtaining

$$I = 4(34.5) + 1(1) = \$139.$$

6. John's utility function is $u(x, y) = 3x + 4y^{1/2}$, his income is $I = \$220$, and $p_y = \$1$. The price of good x decreases from $p_x = \$3$ to $p'_x = \$2$. UFind the substitution and income effect from this price change.

- To begin our analysis, it is useful to consider John's marginal rate of substitution,

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{3}{2y^{-1/2}} = \frac{3}{2}y^{1/2}$$

- *Finding initial bundle* A. Using John's initial prices, we can establish his tangency condition, $\frac{MU_x}{MU_y} = \frac{3}{1}$, as $\frac{3}{2}y^{1/2} = 3$, which simplifies to $y^{1/2} = 2$. Squaring both sides, we can solve for y, to obtain

$$y = (2)^2 = 4 \text{ units.}$$

  Using John's budget line, we can substitute this result in to obtain $3x+1(4) = 220$. Solving this expression for x, we obtain John's initial consumption of good x,

$$x = \frac{220 - 4}{3} = 72 \text{ units.}$$

  This gives us John's initial bundle $A = (72, 4)$.

- *Finding final bundle* C. Now with John's final prices, his tangency condition, $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$ is $\frac{3}{2}y^{1/2} = \frac{2}{1}$, which simplifies to $y^{1/2} = \frac{4}{3}$. Once again, we can square both sides to solve this expression for y to obtain

$$y = \left(\frac{4}{3}\right)^2 = \frac{16}{9} \simeq 1.78 \text{ units.}$$

As before, we use John's budget line and substitute this result in to obtain

$$2x + 1(1.78) = 220.$$

Solving this expression for x, we obtain John's final consumption of good x,

$$x = \frac{220 - 1.78}{2} = 109.11 \text{ units.}$$

which provides John's final bundle $C = (109.11, 1.78)$.

- *Finding decomposition bundle* B. Now we need to determine how John's bundle changes from his initial bundle to his final bundle, breaking them into the substitution and income effects. To calculate this, we need to satisfy both conditions as explained in example 4.9.
  - First, the decomposition bundle must reach the utility level John received under his initial bundle, which we can calculate by inserting bundle A into John's utility function,

    $$3(72) + 4(4)^{1/2} = 224$$

    Thus, John's decomposition bundle B must satisfy $3x + 4y^{1/2} = 224$.
  - Second, the decomposition bundle must be tangent to John's indifference curve. This happens where the slope of John's indifference curve is equal to the slope of his final budget line, which implies that we use the tangency condition from John's final bundle, $y = 1.78$ units.

  Substituting John's tangency condition into the first condition gives

  $$3x + 4(1.78)^{1/2} = 224$$

  We can solve this expression for good x to obtain,

  $$x = \frac{224 - 5.34}{3} = 72.89 \text{ units.}$$

  This gives us John's decomposition bundle $B = (72.89, 1.78)$.

- Lastly, we can calculate John's substitution and income effects by comparing the values of his different bundles:

  $$\text{Substitution Effect:} \quad x_B - x_A = 72.89 - 72 = 0.89 \text{ units}$$
  $$\text{Income Effect:} \quad x_C - x_B = 109.11 - 72.89 = 36.22 \text{ units.}$$

  This result for John should make sense. Since the majority of his utility comes from consuming good x (the quasilinear parameter), John allocates the bulk of his savings from the price change to purchasing more of good x. This leads to a quite large income effect, with a small substitution effect.