Recitation #6 [10/31/2025]

1. Consider a competitive market in which the government will be imposing an ad volorem tax of τ. Aggregate demand curve is $x(p) = Ap^{\varepsilon}$, where $A > 0$ and $\varepsilon < 0$, and aggregate supply curve is $q(p) = \alpha p^{\gamma}$, where $\alpha > 0$ and $\gamma > 0$. Denote $\kappa = 1 + \tau$, and assume that a partial equilibrium analysis is valid.

   (a) Evaluate how the equilibrium price is affected by a marginal increase in the tax, that is, by a marginal increase in κ.

Imposing the ad valorem tax τ, the aggregate demand curve now becomes

$$x(p, \tau) = A\left[p(\tau)(1+\tau)\right]^{\varepsilon}$$

Whereas, the aggregate supply curve becomes

$$q(p, \tau) = \alpha\left[p(\tau)\right]^{\gamma}$$

At equilibrium, the aggregate supply curve intersects the aggregate demand curve so that the market clears, that is,

$$A\left[p(\tau)(1+\tau)\right]^{\varepsilon} = \alpha\left[p(\tau)\right]^{\gamma}$$

Rearranging, we can express the price $p(\tau)$ that the producers receive in terms of tax rate τ.

$$p(\tau) = \left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}(1+\tau)^{\frac{\varepsilon}{\gamma-\varepsilon}}$$

Differentiating the above expression with respect to τ,

$$p'(\tau) = \frac{\varepsilon}{\gamma - \varepsilon}\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}(1+\tau)^{\frac{2\varepsilon-\gamma}{\gamma-\varepsilon}}$$

We evaluate the first order condition at $\tau = 0$ for a marginal increase in the tax, so that

$$p'(0) = \frac{\varepsilon}{\gamma - \varepsilon}\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}(1+0)^{\frac{2\varepsilon-\gamma}{\gamma-\varepsilon}}$$

so that the tax burden borne by the producers is $p'(0) = \frac{c\varepsilon}{\gamma-\varepsilon}$, where $c \equiv \left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}$ is a constant independent of τ. Essentially, c captures the responsiveness of the equilibrium price to the steepness of the aggregate demand and supply curves. In fact, for a higher A which represents a steeper aggregate demand curve, the price received by the producer is more responsive to a marginal change in the ad valorem tax τ, and the opposite is said for a higher α which represents a steeper aggregate supply curve.

We are also interested in the price paid by the consumers, given by

$$\hat{p}(\tau) \equiv p(\tau)\cdot(1+\tau) = \left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}(1+\tau)^{\frac{\gamma}{\gamma-\varepsilon}}$$

Differentiating the above expression with respect to τ,

$$\hat{p}'(\tau) = \frac{\gamma}{\gamma - \varepsilon}\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}(1+\tau)^{\frac{\varepsilon}{\gamma-\varepsilon}}$$

We evaluate the first order condition at $\tau = 0$ for a marginal increase in the tax, so that

$$\hat{p}'(0) = \frac{\gamma}{\gamma - \varepsilon}\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}(1+0)^{\frac{\varepsilon}{\gamma-\varepsilon}}$$

so that the tax burden borne by the consumers is $\hat{p}'(0) = \frac{c\gamma}{\gamma-\varepsilon}$, where $c \equiv \left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}$.

(b) Describe the incidence of the tax when $\gamma = 0$.

$$p'(\tau = 0|\gamma = 0) = -\left(\frac{A}{\alpha}\right)^{-\frac{1}{\varepsilon}}\frac{\varepsilon}{\varepsilon}$$
$$\hat{p}'(\tau = 0|\gamma = 0) = -\left(\frac{A}{\alpha}\right)^{-\frac{1}{\varepsilon}}\frac{0}{\varepsilon}$$

Therefore, when the supply is perfectly inelastic, as given by $\gamma = 0$, the tax is entirely borne by the producers, and the price received by each one of them is reduced by a factor of $\left(\frac{A}{\alpha}\right)^{-\frac{1}{\varepsilon}}$ on the ad valorem tax τ. Whereas, the price paid by the consumers is unaffected by τ.

(c) What is the tax incidence when, instead, $\varepsilon = 0$?

$$p'(\tau = 0|\varepsilon = 0) = \left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma}}\frac{0}{\gamma}$$
$$\hat{p}'(\tau = 0|\varepsilon = 0) = \left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma}}\frac{\gamma}{\gamma}$$

Therefore, when the demand is perfectly inelastic, as given by $\varepsilon = 0$, the tax is entirely borne by the consumers, and the price paid by each one of them is increased by a factor of $\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma}}$ on the ad valorem tax τ. Whereas, the price received by the producers is unaffected by τ.

(d) What happens when each of these elasticities approach ∞ in absolute value?

$$\lim_{\gamma\to\infty} p'(0) = \lim_{\gamma\to\infty}\left[\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}\frac{\varepsilon}{\gamma-\varepsilon}\right]$$

$$= \left(\lim_{\gamma\to\infty}\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}\right)\cdot\left(\lim_{\gamma\to\infty}\frac{\varepsilon}{\gamma-\varepsilon}\right) \quad \text{by the continuity in the limits}$$

$$= 1\cdot 0 = 0$$

$$\lim_{\gamma\to\infty} \hat{p}'(0) = \lim_{\gamma\to\infty}\left[\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}\frac{\gamma}{\gamma-\varepsilon}\right]$$

$$= \left(\lim_{\gamma\to\infty}\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}\right)\left(\lim_{\gamma\to\infty}\frac{1}{1-\frac{\varepsilon}{\gamma}}\right) \quad \text{by the continuity in the limits}$$

$$= 1\cdot 1 = 1$$

Therefore, when the supply is perfectly elastic, as given by $\gamma\to\infty$, the tax is entirely borne by the consumers by a factor of 1, and the price received by the producers is unaffected by the ad valorem tax τ.

$$\lim_{\varepsilon\to-\infty} p'(0) = \lim_{\varepsilon\to-\infty}\left[\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}\frac{\varepsilon}{\gamma-\varepsilon}\right]$$

$$= \left(\lim_{\varepsilon\to-\infty}\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}\right)\cdot\left(\lim_{\varepsilon\to-\infty}\frac{1}{\frac{\gamma}{\varepsilon}-1}\right) \quad \text{by the continuity in the limits}$$

$$= -1\cdot 1 = 1$$

$$\lim_{\varepsilon\to-\infty} \hat{p}'(0) = \lim_{\varepsilon\to-\infty}\left[\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}\frac{\gamma}{\gamma-\varepsilon}\right]$$

$$= \left(\lim_{\varepsilon\to-\infty}\left(\frac{A}{\alpha}\right)^{\frac{1}{\gamma-\varepsilon}}\right)\left(\lim_{\varepsilon\to-\infty}\frac{\gamma}{\gamma-\varepsilon}\right) \quad \text{by the continuity in the limits}$$

$$= 1\cdot 0 = 0$$

Therefore, when the demand is perfectly elastic, as given by $\varepsilon \to -\infty$, the tax is entirely borne by the producers by a factor of 1, and the price received by the consumers is unaffected by the ad valorem tax τ.

(e) Redo your analysis in part (a) the relative change in price due to ad valorem tax τ. How would you compare your results to the effects of a unit tax t in example 6.3?

The relative change in price that the producers receive is

$$\frac{d \ln p(\tau)}{d\tau} = \frac{p'(\tau)}{p(\tau)} = \frac{\varepsilon}{\gamma - \varepsilon} \frac{1}{1+\tau}$$

We evaluate the above first order condition at $\tau = 0$,

$$\frac{p'(0)}{p(0)} = \frac{\varepsilon}{\gamma - \varepsilon}$$

The relative change in price that the consumers pay is

$$\frac{d \ln \hat{p}(\tau)}{d\tau} = \frac{\hat{p}'(\tau)}{\hat{p}(\tau)} = \frac{\gamma}{\gamma - \varepsilon} \frac{1}{1+\tau}$$

We evaluate the above first order condition at $\tau = 0$,

$$\frac{\hat{p}'(0)}{\hat{p}(0)} = \frac{\gamma}{\gamma - \varepsilon}$$

Therefore, the tax burden borne by the producers and consumers relative to the pre-tax price are $\frac{\varepsilon}{\gamma-\varepsilon}$ and $\frac{\gamma}{\gamma-\varepsilon}$ respectively for an ad valorem tax τ, which coincide with the absolute tax burden borne by them for a unit tax t.