

## Homework #7 - Due on Thursday November 13th

1. Let us consider a market with two firms, Firm  $A$  and Firm  $B$ , producing a homogeneous good. However, Firm  $A$  generates more pollution than Firm  $B$  during the production process as explained below. Firm  $i$ 's marginal production costs are given by  $c_i$  where  $i = \{A, B\}$ , where  $c_B$  is strictly higher than  $c_A$ . In addition, the social welfare function that the regulator uses to set emission fees on these firms is

$$SW = CS + PS + T - Env$$

where  $CS$  is the consumer surplus,  $PS$  is the producer surplus,  $T = t(q_A + q_B)$  is the tax revenue from emission fees on both firms, and  $Env = d_A(q_A)^2 + d_B(q_B)^2$  is the environmental damage from the production of both goods, where  $d_A \geq d_B$ . Finally, the inverse demand function of firm  $i = \{A, B\}$  is

$$p_i(q_i, q_j) = 1 - q_i - q_j \quad \text{where } j = \{A, B\} \text{ and } j \neq i.$$

where  $q_i$  denotes output.

- (a) *No regulation.* Find equilibrium output levels when firms do not face emission fees. Interpret. **[10 Points]**
- (b) *Regulation.* Find equilibrium output levels when firms face any emission fee  $t$ . Interpret. **[10 Points]**
- (c) Identify the socially optimal output level for firm A,  $q_A^{SO}$ , and for firm B,  $q_B^{SO}$ . **[10 Points]**
- (d) Find the socially optimal fees ( $t$ ) that induce firms to produce the socially optimal output levels found in part (c). Assume that  $d_A = 2$  and  $d_B = 1$ , and  $c_B = \frac{1}{4}$  and  $c_A = 0$ . **[20 Points]**
- (e) Now consider the case in which both firms invest in clean technology at a cost  $F > 0$ . This investment reduces the amount of pollution of firm  $A$  in  $\alpha q_A$ , where  $\alpha \in (0, 1)$  and that of firm  $B$  in  $\beta q_B$ , where  $\beta \in (0, 1)$  and  $\alpha \neq \beta$ . That is, when  $\alpha$  (or  $\beta$ ) approaches to zero the firm is able to reduce almost all its emissions. Identify the the socially optimal fees ( $t$ ) that induce firms to produce the socially optimal output levels. Represent it in general terms and then assume that  $d_A = 2$  and  $d_B = 1$ , and  $c_B = \frac{1}{4}$  and  $c_A = 0$ . Compare your results with the socially optimal fees in part (d) and discuss under which conditions is this fee in part (d) higher than that in part (e). **[50 Points]**