# Homework #3 (Due on September 25th, 2025)

1. Assume that you have friend, Leo, who is now retired and lives on a fixed income $w > 0$ which does not adjust to inflation. His expenditure function is

$$e(p_1, p_2, u) = (p_1 + p_2)u,$$

where $p_1, p_2 > 0$ denote initial prices. Suppose that prices of goods 1 and 2 increase to $p_1'$ and $p_2'$, respectively.

   (a) You want to give him a monetary gift so that he will not be affected by the above price increase. How much money should you give him? That is, find his compensating variation (CV).

   (b) Now find his equivalent variation (EV) from the price change, i.e., the change in income needed at initial prices $p_1$ and $p_2$ that would have the same effect on utility as would the change in prices, $p_1'$ and $p_2'$.

   (c) Which is larger in this case, CV or EV?

   (d) Find his Walrasian demand for each good.

   (e) Find his utility function. What is this type of utility function called?

2. Chelsea loves chocolate ($x$) and books ($y$), and her utility from consuming these two goods can be represented by a with quasilinear utility function $u(x, y) = \rho\sqrt{x} + \tau y$, where $\rho, \tau > 0$.

   (a) Find the Walrassian demand of the individual.

   (b) Find the Hicksian demand for goods 1 and 2.

   (c) Assume that Chelsea's wealth is $w = \$500$, and prices are $p_1 = p_2 = \$15$. For simplicity, consider parameters $\rho = 1, \tau = \frac{1}{2}$. Find the AV, CV and EV.

3. Consider an individual with utility function $u(q_1, q_2) = q_1^2 + q_2 - 1$, where $q_1$ ($q_2$) denotes the units of good 1 (good 2, respectively) that this individual consumes. His income level is denoted by $w \in \mathbb{R}_+$, and prices are both strictly positive, i.e., $\mathbf{p} = (p_1, p_2) \in \mathbb{R}^2_{++}$.

   (a) Determine this individual's Walrasian demand, and his associated indirect utility function.

   (b) Determine this individual's Hicksian demand, $h_1(\mathbf{p}, u)$ and $h_2(\mathbf{p}, u)$, and his associated expenditure function, $e(\mathbf{p}, u)$.

Consider now that this individual's income level is $w = 6$, and the initial vector of market prices is $\mathbf{p^0} = (4, 3)$. If both prices increase by 50%, determine:

1. (c) The compensating variation of this price increase. Interpret.

   (d) The change in consumer surplus associated to this price increase. Interpret.

4. Consider a firm with production function $q = \sqrt{z}$, using one input (e.g., labor) to produce one type of output. The price of every unit of input is $w = 8$, and the price of every unit of output is $p > 0$.

   (a) Set up the firm's profit-maximization problem, and solve for its unconditional factor demand $z(8, p)$.

   (b) Evaluate the profit function at the unconditional factor demand $z(8, p)$. Test for convexity of the profit function in output price $p$.

   (c) Let us now illustrate convexity in output prices by using an alternative approach: (1) evaluate the profit function you found in part (b) at prices $p = 6$, and at $p = 12$. Then, find their convex combination $\alpha\pi(6) + (1 - \alpha)\,\pi(12)$ where $\alpha \in [0, 1]$; (2) evaluate the profit function at the convex combination of the above output prices, that is, $\pi(\alpha 6 + (1 - \alpha)\,12)$. Last, show that the profit function you found in step (1) lies weakly above that found in step (2) for all values of $\alpha$, that is,

$$\alpha\pi(6) + (1 - \alpha)\,\pi(12) \geq \pi(\alpha 6 + (1 - \alpha)\,12).$$