# Homework # 2 EconS501 [Due on Sepetember 11th, 2025]

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1. Let $(\mathcal{B}, C(\cdot))$ be a choice structure where $\mathcal{B}$ includes all non-empty subsets of consumption bundles $X$, i.e., $C(B) \neq \varnothing$ for all sets $B \in \mathcal{B}$. We define the choice rule $C(\cdot)$ to be *distributive* if, for any two sets $B$ and $B'$ in $\mathcal{B}$,

$$C(B) \cap C(B') \neq \varnothing \text{ implies that } C(B) \cap C(B') = C(B \cap B')$$

In words, the elements that the individual decision maker selects both when facing set $B$ and when facing set $B'$, $C(B) \cap C(B')$, coincide with the elements that he would select when confronted with the elements that belong to both sets $B \cap B'$, i.e., $C(B \cap B')$. Show that, if choice rule $C(\cdot)$ is *distributive*, then choice structure $(\mathcal{B}, C(\cdot))$ does not necessarily satisfy the weak axiom of revealed preference. (A counterexample suffices.)

2. Consider an individual with utility function

$$u(x_1, x_2) = \ln x_1 + x_2,$$

where $x_1$ and $x_2$ denote the amounts consumed of non-organic and organic goods, respectively. The prices of these goods are $p_1 > 0$ and $p_2 > 0$, respectively; and this individual's wealth is $w > 0$.

   (a) Find this consumer's uncompensated demand for every good $x_i(p, w)$, where $i = \{1, 2\}$. [For compactness, we use $p$ to denote the price vector $p \equiv (p_1, p_2)$.] Under which conditions the consumer demands positive amounts of both goods? Interpret your results.

   (b) Find the indirect utility function, $v(p, w)$.

   (c) Find this consumer's expenditure function, $e(p, v)$, and her compensated demand for every good $h_i(p, w)$, where $i = \{1, 2\}$.

   (d) Solve parts (a)-(c) of the exercise again, but considering that the consumer's utility function is now $u(x_1, x_2) = (x_1 - a_1)(x_2 - a_2)$, where parameters $a_1$ and $a_2$ are both weakly positive, $a_1, a_2 \geq 0$.

3. Consider a consumer with utility function $u(x_1, x_2, x_3) = x_1 x_2 x_3$, and income $w$.

   (a) Set up the consumer's utility maximization problem and find the Walrasian demands for each good.

(b) Let $x_1 + \frac{p_2}{p_1} x_2 = x_c$ denote the units of a composite good. Set up the consumer's utility maximization problem again, but now in terms of the composite good $x_c$. Find the Walrasian demand function for the composite good $x_c$.

(c) Show that the Walrasian demands you found in parts (a) and (b) are equivalent.

4. Consider a consumer with quasilinear utility function $u(x, y, q) = v(x, q) + y$, where $x$ denotes units of good $x$, $q$ represents its quality, and $y$ reflects the numeraire good (whose price is normalized to 1). The price of good $x$ is $p > 0$, and the consumer's wealth is $w > 0$. Assume that $v_x, v_q > 0$ and $v_{xx} \leq 0$.

   (a) Set up the consumer's utility maximization problem.

   (b) Show that the Walrasian demand $x(p, q)$ is: (1) decreasing in $p$; and (2) increasing in $q$ if $v_{xq} > 0$. Interpret your results.

   (c) Assume in this part of the exercise that $v_{xq} > 0$ so that $\frac{\partial x(p,q)}{\partial q} > 0$. We say that a Walrasian demand $x(p, q)$ is supermodular in $(p, q)$ if the following property holds

$$\underbrace{x(p,q)\frac{\partial^2 x(p,q)}{\partial p \partial q}}_{\text{First term}} - \underbrace{\underbrace{\frac{\partial x(p,q)}{\partial p}}_{(-)\text{ from part (b)}} \quad \underbrace{\frac{\partial x(p,q)}{\partial p}}_{(+)\text{ from part (b)}}}_{\text{Second term, }+} > 0.$$

   From part (b) we know that $\frac{\partial x(p,q)}{\partial p} < 0$ and that $\frac{\partial x(p,q)}{\partial q}$ is positive. Therefore, for Walrasian demand $x(p, q)$ to be supermodularity we only need that the cross-partial $\frac{\partial^2 x(p,q)}{\partial p \partial q}$ is either positive, entailing an unambigous expression above, or not very negative, so the positive second term offsets the potentially negative first term. Show that supermodularity holds if $v_{xx}v_{xq} + x(v_{xxx}v_{xq} - v_{xxq}v_{xx}) < 0$. Interpret your results.

5. Consider utility function $u(x, y)$, where $x$ and $y$ represent the units of two goods. Assume that $u(\cdot)$ is twice continuously differentiable, strictly increasing and concave in both of its arguments, $x$ and $y$. Assuming that the consumer's wealth is given by $w > 0$, and that he faces a price vector $p = (p_x, p_y) >> 0$, denote his indirect utility function as $v(p, w)$.

   (a) Use the indirect utility function $v(p, w)$ to find the consumer willingness to pay for good $y$.

   (b) Identify under which condition is this willingness to pay for good $y$ increasing or decreasing in income, $w$. Interpret.