Applications of Game Theory to Environmental Problems

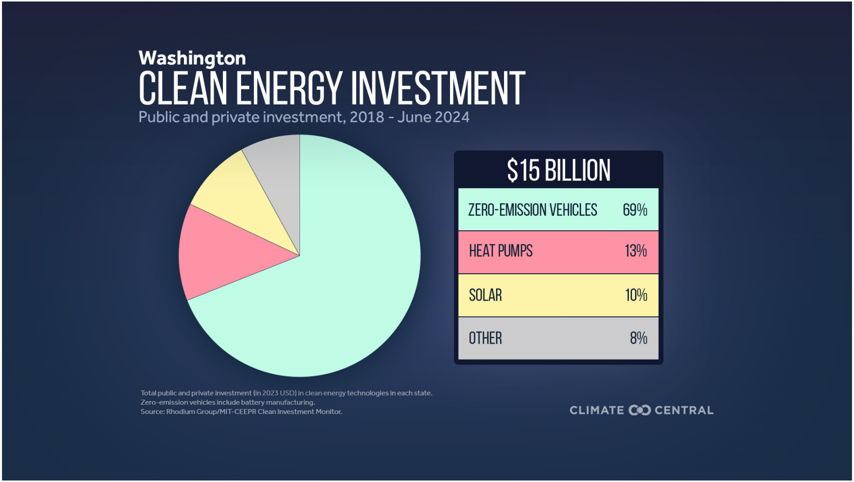
Ana Espinola-Arredondo

July 2025

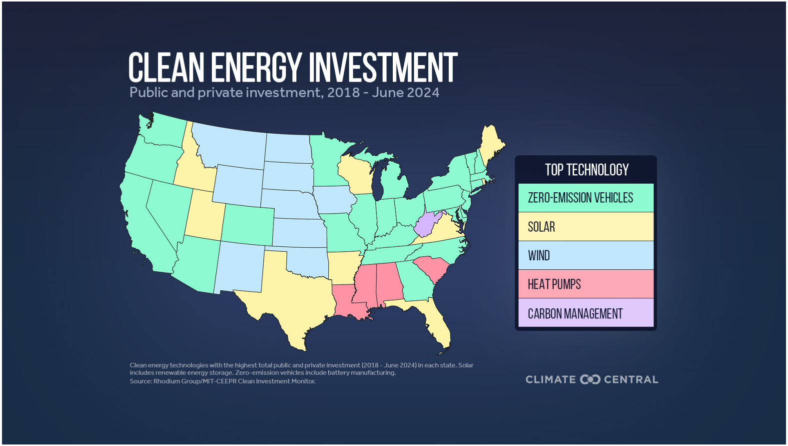
Introduction

- Abatement, R&D and Patents

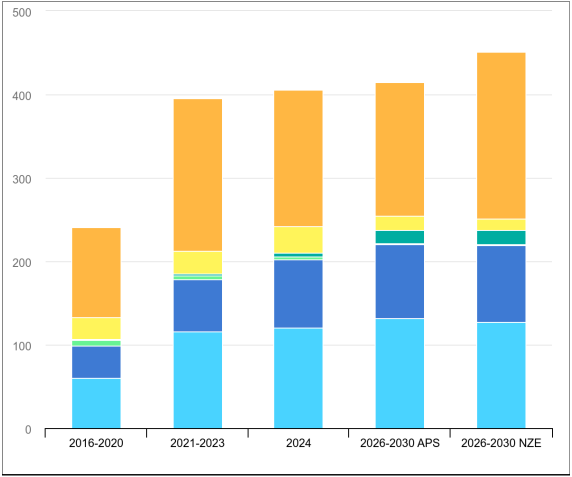
Clean Energy Investment



Clean Energy Investment



Past and future energy investment in the European Union (IEA)

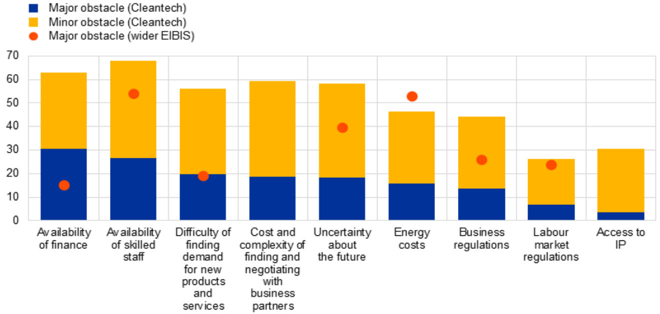


Clean Energy Investment

Chart 7

Obstacles to business activities related to clean and sustainable technologies in the EU

(percentages of firms)

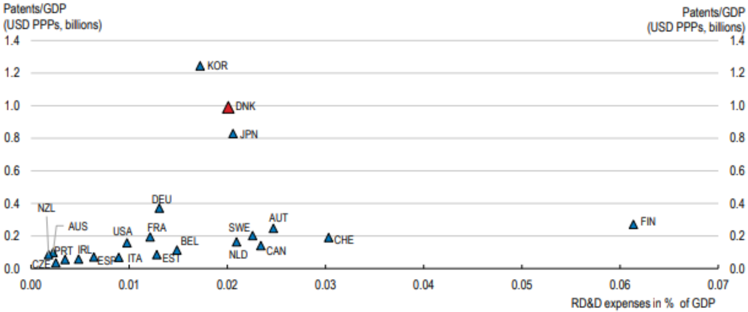


Sources: EPO/EIB (Cleantech Survey) and EIB (EIBIS).

Notes: The EIBIS does not include information on cost and complexity of finding and negotiating with business partners or on access to intellectual property (IP). For details of the Cleantech Survey, see EPO/EIB, op. cit.

Figure 12. High R&D expenses are correlated with many environmentally-related patents

Climate change mitigation technology patents relative to RD&D expenses

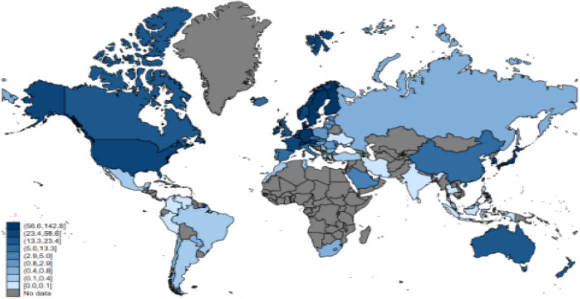


Note: RD&D expenses cover the expenses on energy efficiency (group 1) and renewable energy sources (group 3). Both RD&D expenses and GDP are measured in 2019 USD PPP for 2019. The number of patents is the 2015-2018 average of climate change mitigation technologies related to energy generation, transmission or distribution (family size two and greater), weighted by the 2015-2018 average of the GDP expressed in 2015 USD PPPs billions.

Source: OECD, Environment (Innovation in environment-related technologies - Technology Development) database; OECD, National Accounts database; and IEA, Energy (Energy Technology R&D - RD&D Budget) database.

Figure 5 - Number of patents granted in environmental technologies per million inhabitants, averaged over period 2015-2018

Figure 5 / Number of patents granted in environmental technologies per million inhabitants, averaged over period 2015-2018



Source: Amadeus, authors' calculations, downloaded March 2022.

Source: Amadeus, authors' calculations, downloaded March 2022.

Motivation

- Greener or Cheaper Goods: Economies of Scope in R&D Investments (JEEM, 2024)
- Firms investments in cost-reducing R&D are large and increasing:
  - \$625 billion in the US and \$310 billion in the EU; OECD (2021).
- Environmental R&D (ER&D) has also increased:
  - Investments in low-carbon technologies reached \$755 billion in 2021 (25% increase); Bloomberg.
- Both investments separately received attention, but firms' simultaneous choice of R&D and ER&D has been largely overlooked.
  - Most chemical companies recognize investing in both, Potters and Grassano (2019).

Motivation

- We allow for both investments, helping us:
  - Better understand firms' decisions.
  - Avoid potential regulatory mistakes (undertaxation).

Motivation

- Why not just analyze R&D and ER&D separately?
  - We could...
  - if their marginal benefits and costs were additively separable.
    - A larger investment in one didn't affect firms' incentives to invest in the other.
- But are they separable?

Motivation

- Benefits are likely not separable.
  - If a firm invests in R&D, it lowers its production costs, increasing pollution,
  - This triggers a more stringent emission fee,
  - ultimately increasing firms' incentives to invest in abatement.

Motivation

- Costs may not be separable either:
    - Waterless dying technologies in the textile industry, Heida (2014).
    - Innovations originally developed to reduce emissions can also be used to reduce costs.
    - We refer to them as "economies of scope" in investments:
        - Investing in multiple forms of R&D is less costly than separately investing in each of them.

Motivation

- We also allow for "diseconomies of scope" in investments:
    - Innovations developed to reduce emissions end up increasing costs.
    - Examples abound in firms' green investments in the EU.

Motivation - Regulatory implications

- Ignoring firms' simultaneous investment decisions gives rise to an *undertaxation* problem.
- First, consider no economies of scope.
  - We show that R&D and ER&D are strategic complements...
    - leading to more investments in both.
    - but especially more investments in cost-reducing R&D.
  - Anticipating more production and pollution, the regulator sets a more stringent fee.
  - Ignoring the multiplicity of investments leads to lower-than-optimal fees.
- Second, allowing for economies of scope.

Motivation - Regulatory implications

- Ignoring firms' simultaneous investment decisions gives rise to an *undertaxation* problem.
- First, consider no economies of scope.
  - We show that R&D and ER&D are strategic complements...
    - leading to more investments in both.
    - but especially more investments in cost-reducing R&D.
  - Anticipating more production and pollution, the regulator sets a more stringent fee.
  - Ignoring the multiplicity of investments leads to lower-than-optimal fees.
- Second, allowing for economies of scope.
  - More investment in abatement.

Motivation - Regulatory implications

- Ignoring firms' simultaneous investment decisions gives rise to an *undertaxation* problem.
- First, consider no economies of scope.
    - We show that R&D and ER&D are strategic complements...
        - leading to more investments in both.
        - but especially more investments in cost-reducing R&D.
    - Anticipating more production and pollution, the regulator sets a more stringent fee.
    - Ignoring the multiplicity of investments leads to lower-than-optimal fees.
- Second, allowing for economies of scope.
    - More investment in abatement.
    - The reg. can set less stringent fees...

Motivation - Regulatory implications

- Ignoring firms' simultaneous investment decisions gives rise to an *undertaxation* problem.
- First, consider no economies of scope.
  - We show that R&D and ER&D are strategic complements...
    - leading to more investments in both.
    - but especially more investments in cost-reducing R&D.
  - Anticipating more production and pollution, the regulator sets a more stringent fee.
  - Ignoring the multiplicity of investments leads to lower-than-optimal fees.
- Second, allowing for economies of scope.
  - More investment in abatement.
  - The reg. can set less stringent fees...
  - Ameliorating undertaxation.

Literature-I

- We contribute to three branches:
- Firms' abatement decisions.
    - Poyago-Theotoky (2007), Montero (2011), Lambertini et al. (2017), and Strandholm et al. (2018, 2023), among others.
    - We allow for both investments, and how emission fees are affected.
    - Petrakis and Poyago-Theotoky (2002) study subsidies in a model with both types of investments, but assume an exogenous emission fee (no economies of scope).
- Cost-reducing R&D.

Literature-I

- We contribute to three branches:
- Firms' abatement decisions.
    - Poyago-Theotoky (2007), Montero (2011), Lambertini et al. (2017), and Strandholm et al. (2018, 2023), among others.
    - We allow for both investments, and how emission fees are affected.
    - Petrakis and Poyago-Theotoky (2002) study subsidies in a model with both types of investments, but assume an exogenous emission fee (no economies of scope).
- Cost-reducing R&D.
    - Seminal article by d'Aspremont and Jacquemin (1988), followed by Kamien et al. (1992), and Matsumura et al. (2013).

Literature-I

- We contribute to three branches:
- Firms' abatement decisions.
  - Poyago-Theotoky (2007), Montero (2011), Lambertini et al. (2017), and Strandholm et al. (2018, 2023), among others.
  - We allow for both investments, and how emission fees are affected.
  - Petrakis and Poyago-Theotoky (2002) study subsidies in a model with both types of investments, but assume an exogenous emission fee (no economies of scope).
- Cost-reducing R&D.
  - Seminal article by d'Aspremont and Jacquemin (1988), followed by Kamien et al. (1992), and Matsumura et al. (2013).
  - We show that their results can underestimate firms' investment in R&D.

Literature-II

- Investing in abatement because of CSR reasons.
  - Baron (2001, 2008), Farzin (2003), and Calveras and Ganuza (2006), among others.
  - Alternative channel for green investment
    - Without having to rely on green consumers.
    - Even in the absence of environmental regulation.

Outline of the presentation

- Model
- Equilibrium behavior.
  - Unregulated oligopoly (benchmark).
  - Regulated oligopoly
- Comparing investments with and without regulation.

Outline of the presentation

- Model
- Equilibrium behavior.
  - Unregulated oligopoly (benchmark).
  - Regulated oligopoly
- Comparing investments with and without regulation.
- Comparing the equilibrium fee against that in standard models.

Outline of the presentation

- Model
- Equilibrium behavior.
  - Unregulated oligopoly (benchmark).
  - Regulated oligopoly
- Comparing investments with and without regulation.
- Comparing the equilibrium fee against that in standard models.
- Sequential investments:

Outline of the presentation

- Model
- Equilibrium behavior.
  - Unregulated oligopoly (benchmark).
  - Regulated oligopoly
- Comparing investments with and without regulation.
- Comparing the equilibrium fee against that in standard models.
- Sequential investments:
  - First, invest in abatement; then, in R&D.

Outline of the presentation

- Model
- Equilibrium behavior.
  - Unregulated oligopoly (benchmark).
  - Regulated oligopoly
- Comparing investments with and without regulation.
- Comparing the equilibrium fee against that in standard models.
- Sequential investments:
  - First, invest in abatement; then, in R&D.
  - Is regulation more effective under sim. or seq. investments?

Model

- **Time structure:**
  - Stage 1. Every firm i chooses its investment in cost-reducing R&D, k_i, and ER&D, z_i.
  - Stage 2. The regulator responds with emission fee $t \geq 0$.
  - Stage 3. Firms compete à la Cournot.
- $n \geq 2$ firms facing inverse demand function $p(Q) = 1 - Q$, where $Q \geq 0$ denotes aggregate output.

Model

- **Time structure:**
  - Stage 1. Every firm i chooses its investment in cost-reducing R&D, k_i, and ER&D, z_i.
  - Stage 2. The regulator responds with emission fee $t \geq 0$.
  - Stage 3. Firms compete à la Cournot.
- $n \geq 2$ firms facing inverse demand function $p(Q) = 1 - Q$, where $Q \geq 0$ denotes aggregate output.
- Marginal cost, $c \in [0, 1]$, decreases to $c - k_i$.

Model

- **Time structure:**
  - Stage 1. Every firm i chooses its investment in cost-reducing R&D, k_i, and ER&D, z_i.
  - Stage 2. The regulator responds with emission fee $t \geq 0$.
  - Stage 3. Firms compete à la Cournot.
- $n \geq 2$ firms facing inverse demand function $p(Q) = 1 - Q$, where $Q \geq 0$ denotes aggregate output.
- Marginal cost, $c \in [0, 1]$, decreases to $c - k_i$.
- Emissions from firm i are $e = q_i - z_i$.

Model

- Total investment cost is

$$C(k_i, z_i) = \frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda k_i z_i,$$

where γ and α denote the efficiency in R&D and ER&D, respectively.

- Marginal costs: $C_{k_i} = \gamma k_i - \lambda z_i$ and $C_{z_i} = \alpha z_i - \lambda k_i$.
- Special cases:

Model

- Total investment cost is

$$C(k_i, z_i) = \frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda k_i z_i,$$

  where γ and α denote the efficiency in R&D and ER&D, respectively.
- Marginal costs: $C_{k_i} = \gamma k_i - \lambda z_i$ and $C_{z_i} = \alpha z_i - \lambda k_i$.
- Special cases:
  - If $k_i = 0$, total cost simplifies to $\frac{1}{2}\alpha z_i^2$ [Poyago-Theotoky (2007)].

Model

- Total investment cost is

$$C(k_i, z_i) = \frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda k_i z_i,$$

  where γ and α denote the efficiency in R&D and ER&D, respectively.
- Marginal costs: $C_{k_i} = \gamma k_i - \lambda z_i$ and $C_{z_i} = \alpha z_i - \lambda k_i$.
- Special cases:
  - If $k_i = 0$, total cost simplifies to $\frac{1}{2}\alpha z_i^2$ [Poyago-Theotoky (2007)].
  - If $z_i = 0$, total cost collapses to $\frac{1}{2}\gamma k_i^2$ [Traditional R&D models].

Model

- Economies of scope:
  - If $\lambda = 0$, total costs in R&D and ER&D are independent.
  - If $\lambda > 0$ ($\lambda < 0$) economies (diseconomies) of scope arise.
- Assumption I (γ and α are high enough, $\gamma, \alpha \geq \frac{2n^2}{(n+1)^2}$)
- Assumption II ($\lambda < \overline{\lambda} \equiv \frac{\sqrt{\alpha(\gamma(n+1)^2 - 2n)}}{(n+1)}$ is not excessive).

Benchmark - No regulation

Unregulated Oligopoly

Benchmark - No regulation

- **Last stage:**
  - Every firm i takes $(k_1, ..., k_n)$ and $(z_1, ..., z_n)$ as given, and solves
    $$\max_{q_i \geq 0} \ (1 - q_i + Q_{-i})q_i - (c - k_i)q_i$$
    where $Q_{-i} = \sum_{j \neq i} q_j$.
  - Cournot model with n cost-asymmetric firms:
    - The investment profile $(k_1, \ldots, k_n)$ can entail a different net production cost $c - k_i$ for each firm i.

Benchmark - No regulation

- **Lemma 1 (summary)**
  - Equilibrium output is $q_i^{NR} = \frac{1-c+nk_i-K_{-i}}{n+1}$, which increases in k_i, but decreases in c, n, and $K_{-i} = \sum_{j \neq i} k_j$;
- Therefore every firm's output increases in its cost advantage, either because:
  - its own R&D investment k_i is higher, or
  - its rivals' investment K_{-i} is lower.

Benchmark - No regulation

- **First stage:**
  - In the first stage, each firm i anticipates profit $\pi_i^{NR} = (q_i^{NR})^2$ and solves

$$\max_{k_i, z_i \geq 0} \ \frac{(1 - c + nk_i - K_{-i})^2}{(n+1)^2} - \left(\frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda k_i z_i \right).$$

  Differentiating with respect to k_i, yields best response function

$$k_i(K_{-i}) = \frac{2n(1-c) + \lambda(n+1)^2 z_i}{\gamma + n[(\gamma - 2)n + 2\gamma]} - \frac{2n}{\gamma + n[(\gamma - 2)n + 2\gamma]} K_{-i}.$$

- Because of Assumption I, $k_i(K_{-i})$ originates in the positive quadrant and decreases in K_{-i} (strategic substitutes).

Benchmark - No regulation

- **First stage:**
- Special cases:
  - When $\lambda = 0$, this best response function simplifies to

$$k_i(K_{-i}) = \frac{2n(1-c)}{\gamma + n[(\gamma - 2)n + 2\gamma]} - \frac{2n}{\gamma + n[(\gamma - 2)n + 2\gamma]} K_{-i},$$

    meaning that abatement decisions, z_i, do not affect R&D investment.
  - When $\lambda > 0$, however, $k_i(K_{-i})$ shifts upward, without changing its slope:
    - Firms have stronger incentives to invest in R&D, but its strategic substitutability is unaffected.
- What about abatement?

Benchmark - No regulation

- **First stage:**
- Special cases:
    - When $\lambda = 0$, this best response function simplifies to

      $$k_i(K_{-i}) = \frac{2n(1-c)}{\gamma + n[(\gamma - 2)n + 2\gamma]} - \frac{2n}{\gamma + n[(\gamma - 2)n + 2\gamma]} K_{-i},$$

      meaning that abatement decisions, z_i, do not affect R&D investment.
    - When $\lambda > 0$, however, $k_i(K_{-i})$ shifts upward, without changing its slope:
        - Firms have stronger incentives to invest in R&D, but its strategic substitutability is unaffected.
- What about abatement?
    - Differentiating with respect to z_i, yields $z_i = \frac{\lambda}{\alpha} k_i$.

Benchmark - No regulation

- **Lemma 2 (summary)**
- When $\lambda \leq 0$, corner solution where $z_i^{NR} = 0$ and $k_i^{NR} > 0$.
  - As in the models where firms can only invest in abatement.
  - No abatement without regulation.
- When $\lambda > 0$, we find an interior solution:

Benchmark - No regulation

- **Lemma 2 (summary)**
- When $\lambda \leq 0$, corner solution where $z_i^{NR} = 0$ and $k_i^{NR} > 0$.
  - As in the models where firms can only invest in abatement.
  - No abatement without regulation.
- When $\lambda > 0$, we find an interior solution:
  - $z_i^{NR} = \frac{2(1-c)\lambda n}{(n+1)^2(\alpha\gamma-\lambda^2)-2\alpha n}$ and $k_i^{NR} = \frac{2\alpha(1-c)n}{(n+1)^2(\alpha\gamma-\lambda^2)-2\alpha n}$, both positive.

Benchmark - No regulation

- **Lemma 2 (summary)**
- When $\lambda \leq 0$, corner solution where $z_i^{NR} = 0$ and $k_i^{NR} > 0$.
  - As in the models where firms can only invest in abatement.
  - No abatement without regulation.
- When $\lambda > 0$, we find an interior solution:
  - $z_i^{NR} = \frac{2(1-c)\lambda n}{(n+1)^2(\alpha\gamma-\lambda^2)-2\alpha n}$ and $k_i^{NR} = \frac{2\alpha(1-c)n}{(n+1)^2(\alpha\gamma-\lambda^2)-2\alpha n}$, both positive.
  - Despite the absence of emission fees, investments lower each other's marginal costs.

Introducing Environmental Regulation

Introducing environmental regulation

- **Third stage:**
  - Observing the investment profile $(k_1, \ldots, k_n, z_1, \ldots, z_n)$ and emission fee t, every firm i solves

$$\max_{q_i \geq 0} (1 - q_i - Q_{-i})q_i - (c - k_i)q_i - t(q_i - z_i)$$

which yields output $q_i^R = \frac{1-c-t+nk_i-K_{-i}}{n+1}$, with associated profit $\pi_i^R = (q_i^R)^2 + tz_i$.

Introducing environmental regulation

- **Second stage:**
  - The regulator chooses t to maximize

$$\max_{t} \quad SW = CS(Q) + PS(Q) + T(Q) - ED(Q)$$

    where $CS(Q) = \frac{1}{2}Q^2$ denotes consumer surplus,
  - $PS(Q)$ represents aggregate profits net of taxes,
  - $T(Q) = t(Q - Z)$ denotes total tax collection, with Z denoting aggregate abatement, and
  - $ED(Q) = d(Q - Z)^2$ measures aggregate environmental damages, where $d > 1$ denotes pollution severity.
- Aggregate output is evaluated at $Q^R = nq_i^R$.
- The optimal emission fee is the following.

Introducing environmental regulation

- **Second stage:**
  - **Lemma 3.** The emission fee t is

    $$t(K,Z) = \frac{(2dn-1)[K+n(1-c)] - dn(n+1)Z}{(2d+1)n^2}$$

    which is increasing in environmental damage, d, and R&D, K, but decreasing in ER&D, Z, and marginal cost, c.
  - In addition, the fee is positive if and only if $d > d(n) \equiv \frac{(1-c)n+K}{2n[n(1-c-Z)+K-Z]}$, where cutoff $d(n)$ is increasing in c and Z, but decreasing in K.
- Increase in z_i lowers the stringency of $t(K,Z)$; positive externality.

Introducing environmental regulation

- **Second stage:**
  - **Lemma 3.** The emission fee t is
    $$t(K, Z) = \frac{(2dn - 1)[K + n(1 - c)] - dn(n + 1)Z}{(2d + 1)n^2}$$
    which is increasing in environmental damage, d, and R&D, K, but decreasing in ER&D, Z, and marginal cost, c.
  - In addition, the fee is positive if and only if $d > d(n) \equiv \frac{(1-c)n+K}{2n[n(1-c-Z)+K-Z]}$, where cutoff $d(n)$ is increasing in c and Z, but decreasing in K.
- Increase in z_i lowers the stringency of $t(K, Z)$; positive externality.
- Increase in k_i increases this stringency; negative externality (novel in this literature).

Introducing environmental regulation

- **Second stage:**
  - **Lemma 3.** The emission fee t is
  $$t(K,Z) = \frac{(2dn-1)[K+n(1-c)] - dn(n+1)Z}{(2d+1)n^2}$$
  which is increasing in environmental damage, d, and R&D, K, but decreasing in ER&D, Z, and marginal cost, c.
  - In addition, the fee is positive if and only if $d > d(n) \equiv \frac{(1-c)n+K}{2n[n(1-c-Z)+K-Z]}$, where cutoff $d(n)$ is increasing in c and Z, but decreasing in K.
- Increase in z_i lowers the stringency of $t(K,Z)$; positive externality.
- Increase in k_i increases this stringency; negative externality (novel in this literature).
- In addition, $t(K,Z)$ is separable in K and Z, $\frac{\partial^2 t(K,Z)}{\partial Z \partial K} = 0$, i.e., no cross effects.

Introducing environmental regulation

- **First stage:**
  - Anticipating fee $t(K,Z)$, every firm i solves

$$\max_{k_i, z_i > 0} \left(\frac{1 - c - t(K,Z) + nk_i - K_{-i}}{n+1} \right)^2 + t(K,Z) z_i - \left[\frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda(k_i z_i) \right]$$

- Differentiating with respect to k_i and z_i yields $k_i(z_i, Z_{-i})$ and $z_i(k_i, K_{-i})$.
  - Each form of investment is increasing in the other type, which holds even when $\lambda = 0$
  - k_i and z_i are, then, strategic complements.

Introducing environmental regulation

- **k_i and z_i are strategic complements.**
- *Intuition:*
- A larger abatement induces a less stringent emission fee,
  - allowing firms to invest more in R&D.
- Similarly, a larger investment in R&D triggers a more stringent emission fee,

Introducing environmental regulation

- **k_i and z_i are strategic complements.**
- *Intuition:*
- A larger abatement induces a less stringent emission fee,
  - allowing firms to invest more in R&D.
- Similarly, a larger investment in R&D triggers a more stringent emission fee,
  - which induces firms to invest more in abatement.

Introducing environmental regulation

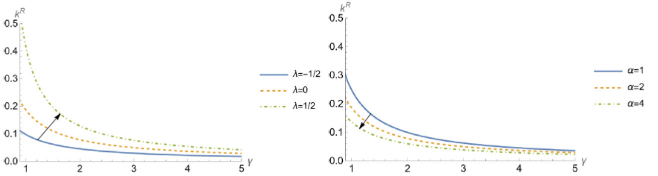
- **k_i and z_i are strategic complements.**
- *Intuition:*
- A larger abatement induces a less stringent emission fee,
  - allowing firms to invest more in R&D.
- Similarly, a larger investment in R&D triggers a more stringent emission fee,
  - which induces firms to invest more in abatement.
- This complementarity provides firms with more incentives to invest in both forms of R&D

Introducing environmental regulation

- **k_i and z_i are strategic complements.**
- *Intuition:*
- A larger abatement induces a less stringent emission fee,
  - allowing firms to invest more in R&D.
- Similarly, a larger investment in R&D triggers a more stringent emission fee,
  - which induces firms to invest more in abatement.
- This complementarity provides firms with more incentives to invest in both forms of R&D
  - than in models that consider a single type of investment, as we next show.

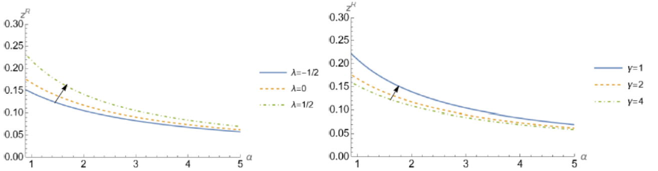
Introducing environmental regulation

- Equilibrium k^R and z^R.



(a) Equilibrium R&D k^R at different levels of λ. (b) Equilibrium R&D k^R at different levels of α.

Figure 1: Comparative statics for equilibrium R&D k^R.



(a) Equilibrium ER&D z^R at different levels of λ. (b) Equilibrium ER&D z^R at different levels of γ.

Figure 2: Comparative statics for equilibrium ER&D z^R.

Introducing environmental regulation

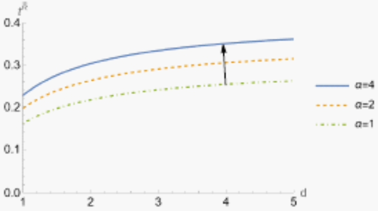
- *The Equilibrium emission fee is*

$$t^R = \frac{1}{C}(1-c)\left[\begin{array}{c} 2d^2n^3(2\alpha\gamma+\gamma-2\lambda(\lambda+1)) \\ -dn(\gamma(1-2\alpha(n-1)n+n) \\ +2\lambda((\lambda+1)(n-1)n-1)) \\ -\lambda-n^2(\alpha\gamma-\lambda^2)] \end{array}\right]$$

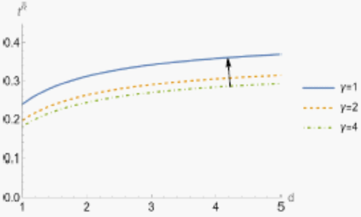
When $\gamma \to \infty$, the equilibrium emission fee simplifies to $\bar{t}^R = \frac{(1-c)[d(2dn^2-n-1)+\alpha(2d+1)n(2dn-1)]}{n[\alpha(2d+1)^2 n+d(2d+1)n(n+2)+d]}$, *and when $\alpha \to \infty$, the fee becomes* $t^R = \frac{(1-c)\gamma(2d+1)n^2(2dn-1)}{(2d+1)n[n(\gamma n(2d+1)-2)+2]-2}$.

Introducing environmental regulation

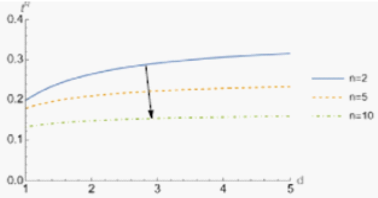
-



(a) Equilibrium fee t^R at different levels of α.



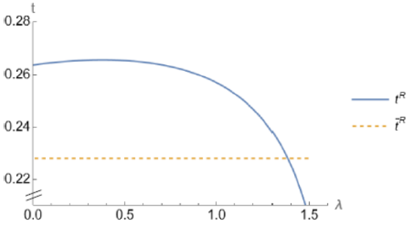
(b) Equilibrium fee t^R at different levels of γ.



(c) Equilibrium fee t^R at different levels of n.

Introducing environmental regulation

- **Effect of regulation:**



where $\bar{t}^R = \lim_{\gamma \to +\infty} t^R$, and coincides with that in PT (2007).

-
  - Undertaxation for most values of λ.
  - Ameliorated when the market is more competitive, pollution is not severe, and investments are more expensive.

Introducing environmental regulation

- **Effect of regulation:**
  - Increases abatement, $z^R > z^{NR}$.
  - Decreases R&D, $k^R < k^{NR}$
- This happens regardless of eco. of scope, λ.
- It is emphasized when investments are more costly (higher γ or α).

Comparison with models assuming separable investments

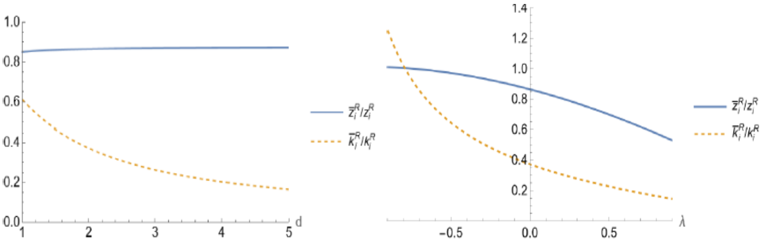
Introducing environmental regulation

- **Investment ratios:**
  - Let $\overline{z}^R = \lim_{\gamma \to +\infty} z^R$, as in Poyago-Theotoky (2007); and $\overline{k}^R = \lim_{\alpha \to +\infty} k^R$, as in standard R&D models.
  - Then, define investment ratios

$$\frac{\overline{z}^R}{z^R} \text{ and } \frac{\overline{k}^R}{k^R}.$$

- *Interpretation:*
  - If close to 1, we are "not losing much" by assuming separable investments
    - We could consider $\overline{z}^R$ and $\overline{k}^R$ to design policy, instead of z^R and k^R.
  - If lower than 1, we are underestimating investments.
  - If higher than 1, we are overestimating investments.

Introducing environmental regulation



(a) Ratios $\overline{z}_i^R/z_i^R$ and $\overline{k}_i^R/k_i^R$ as a function of d. (b) Ratios $\overline{z}_i^R/z_i^R$ and $\overline{k}_i^R/k_i^R$ as a function of λ.

Extension: Sequential investment decisions

Sequential investments - No regulation

- **Third stage.** Output decisions coincide with Lemma 1.
- **Second stage.** R&D decisions, taking abatement $(z_1, ..., z_n)$ as given:
  - $k_i^{NR,Seq}(z_i) = \frac{2(1-c)n+(n+1)^2\lambda z_i}{\gamma(n+1)^2-2n}$, which is positive and increasing in z_i and λ.
- Therefore, when either abatement is nil, $z_i = 0$, or economies of scope are absent, $\lambda = 0$...
  - R&D investment simplifies to $k_i^{NR,Seq}(0) = \frac{2(1-c)n}{\gamma(n+1)^2-2n}$, coinciding with Lemma 2.
- However, when both abatement and economies of scope are positive, $z_i > 0$ and $\lambda > 0$,

Sequential investments - No regulation

- **Third stage.** Output decisions coincide with Lemma 1.
- **Second stage.** R&D decisions, taking abatement $(z_1, ..., z_n)$ as given:
  - $k_i^{NR,Seq}(z_i) = \frac{2(1-c)n+(n+1)^2\lambda z_i}{\gamma(n+1)^2-2n}$, which is positive and increasing in z_i and λ.
- Therefore, when either abatement is nil, $z_i = 0$, or economies of scope are absent, $\lambda = 0$...
  - R&D investment simplifies to $k_i^{NR,Seq}(0) = \frac{2(1-c)n}{\gamma(n+1)^2-2n}$, coinciding with Lemma 2.
- However, when both abatement and economies of scope are positive, $z_i > 0$ and $\lambda > 0$,
  - Every firm benefits from lower marginal costs in its traditional R&D,

Sequential investments - No regulation

- **Third stage.** Output decisions coincide with Lemma 1.
- **Second stage.** R&D decisions, taking abatement $(z_1, ..., z_n)$ as given:
  - $k_i^{NR,Seq}(z_i) = \frac{2(1-c)n+(n+1)^2\lambda z_i}{\gamma(n+1)^2-2n}$, which is positive and increasing in z_i and λ.
- Therefore, when either abatement is nil, $z_i = 0$, or economies of scope are absent, $\lambda = 0$...
  - R&D investment simplifies to $k_i^{NR,Seq}(0) = \frac{2(1-c)n}{\gamma(n+1)^2-2n}$, coinciding with Lemma 2.
- However, when both abatement and economies of scope are positive, $z_i > 0$ and $\lambda > 0$,
  - Every firm benefits from lower marginal costs in its traditional R&D,
  - responding by increasing this investment.

Sequential investments - No regulation

- **First stage.** Anticipating $k_i^{NR,Seq}(z_i)$, every firm solves

$$\max_{z_i \geq 0} \frac{(1-c+k_i^{NR,Seq}(z_i))^2}{(n+1)^2} - \left(\frac{1}{2}\gamma\left(k_i^{NR,Seq}(z_i)\right)^2 + \frac{1}{2}\alpha z_i^2 - \lambda\left(k_i^{NR,Seq}(z_i)\right) z_i\right).$$

yielding the first-order condition

$$\underbrace{\frac{\partial k_i^{NR,Seq}(z_i)}{\partial z_i}}_{(+,\text{ Strategic effect})} \underbrace{\left[\frac{2(1-c+k_i^{NR,Seq}(z_i))}{(n+1)^2} - (\gamma k_i^{NR,Seq}(z_i) - \lambda z_i)\right]}_{(-)}$$

$$+\alpha z_i - \lambda k_i^{NR,Seq}(z_i) = 0,$$

- Relative to sim., seq. decisions give rise to a new strategic effect.

Sequential investments - No regulation

- **First stage.**
- *Simultaneous setting*:
  - An increase in abatement lowers R&D costs if $\lambda > 0$;
  - helping firms increase their R&D investments which, in turn, reduces abatement costs;
  - Positive “*feedback*" effect.
- *Sequential investments*: break this feedback.

Sequential investments - No regulation

- **First stage.**
- *Simultaneous setting*:
  - An increase in abatement lowers R&D costs if $\lambda > 0$;
  - helping firms increase their R&D investments which, in turn, reduces abatement costs;
  - Positive "*feedback*" effect.
- *Sequential investments*: break this feedback.
  - An increase in abatement still lowers R&D costs, inducing more R&D investments, but...

Sequential investments - No regulation

- **First stage.**
- *Simultaneous setting*:
  - An increase in abatement lowers R&D costs if $\lambda > 0$;
  - helping firms increase their R&D investments which, in turn, reduces abatement costs;
  - Positive “*feedback*" effect.
- *Sequential investments*: break this feedback.
  - An increase in abatement still lowers R&D costs, inducing more R&D investments, but...
  - an increase in R&D does not affect abatement costs because abatement decisions are now taken as given in the second stage.

Sequential investments - No regulation

- **First stage.**
- *Simultaneous setting*:
  - An increase in abatement lowers R&D costs if $\lambda > 0$;
  - helping firms increase their R&D investments which, in turn, reduces abatement costs;
  - Positive "*feedback*" effect.
- *Sequential investments*: break this feedback.
  - An increase in abatement still lowers R&D costs, inducing more R&D investments, but...
  - an increase in R&D does not affect abatement costs because abatement decisions are now taken as given in the second stage.
  - Hence, $z_i^{NR,Seq} < z_i^{NR}$.

Sequential investments - No regulation

- **First stage.**
- **Proposition 2.** In the sequential-investment game without regulation, equilibrium investments satisfy:
  - If $\lambda = 0$, equilibrium abatement is $z_i^{NR,Seq} = 0$, yielding an equilibrium R&D of $k_i^{NR,Seq} = \frac{2(1-c)n}{\gamma(n+1)^2-2n}$.
  - If $\lambda > 0$, equilibrium abatement is $z_i^{NR,Seq} = \frac{2\lambda(1-c)\left[n^2(\gamma-2)+\gamma(2n+1)\right]}{D}$, yielding an equilibrium R&D of $k_i^{NR,Seq} = \frac{2(1-c)[nD+\lambda^2(n+1)^2(n^2(\gamma-2)+\gamma(2n+1))]}{D[(n+1)^2\gamma-2n]}$.
- Role of sequential investments?

Sequential investments - No regulation

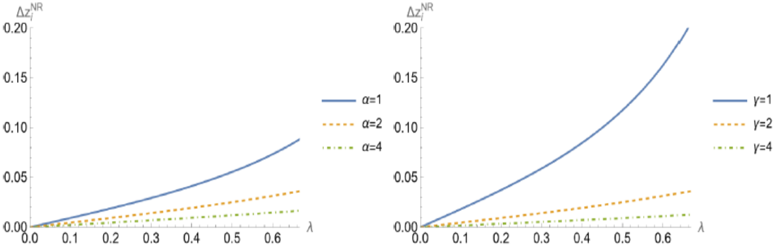
- **Measuring the effect of sequential investments.**
- Abatement differential

$$\Delta z_i^{NR} \equiv z_i^{NR} - z_i^{NR,Seq},$$

  - where z_i^{NR} from Lemma 2 and $z_i^{NR,Seq}$ in Proposition 2.
- When $\Delta z_i^{NR} > 0$, firms invest more in abatement when their R&D decisions are sim than seq;
  - the opposite applies when $\Delta z_i^{NR} < 0$.

Sequential investments - No regulation

- When $\lambda = 0$, abatement coincides, $\Delta z_i^{NR} = 0$.
- When $\lambda > 0$, abatement is larger under sim, $\Delta z_i^{NR} > 0$ (feedback effect).



(a) Differential Δz_i^{NR} at different values of α. (b) Differential Δz_i^{NR} at different values of γ.

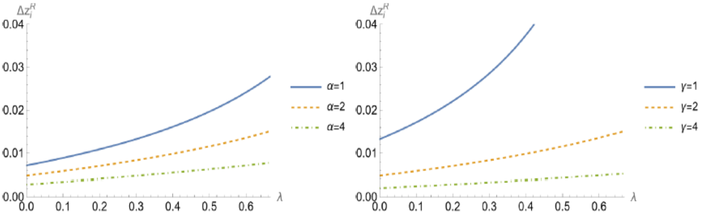
Figure 8: Differential Δz_i^{NR} over a range of λ.

Sequential investments - Regulation

- Similar approach with regulation, but:
- The strategic effect can now be positive or negative.
- *Interpretation*:
  - As under no reg., no feedback effect: less incentives to invest in z_i relative to sim.
  - Reg. gives rise to a second effect: investing in z_i to lower emissions, yielding less stringent fee.
  - Under simultaneous investments, firms face strategic uncertainty about the $(k_1, ..., k_n)$ profile.
  - Under sequential investments, they can anticipate this profile, investing more in z_i to lower production costs.
- If feedback effect dominates, firms would invest less under seq. than sim.

Sequential investments - Regulation

- Abatement differential $\Delta z_i^R \equiv z_i^R - z_i^{R,Seq}$.
  - Feedback effect dominates, $z_i^R > z_i^{R,Seq}$.
  - Even if $\lambda = 0$, unlike without regulation.
  - Increasing in λ, but decreasing in α and γ.



(a) Differential Δz_i^R at different values of α. (b) Differential Δz_i^R at different values of γ.

Figure 9: Differential Δz_i^R over a range of λ.

Sequential investments - Regulation

- **Simultaneous vs. Sequential Fees**
  - Because $z_i^R > z_i^{R,Seq}$, fees respond in opposite direction, $t^{R,Seq} < t^R$.
  - Then, seq. investments emphasize undertaxation problems, $t^{R,Seq} - \overline{t}^R > t^R - \overline{t}^R$.

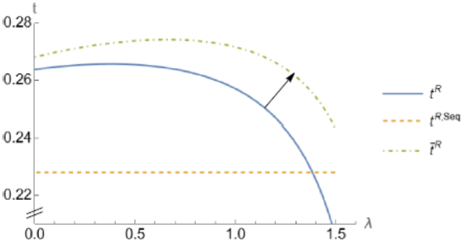


Figure 10: Equilibrium fees $t^{R,Seq}$, t^R, and $\overline{t}^R$ over a range of λ.

Sequential investments - Regulation

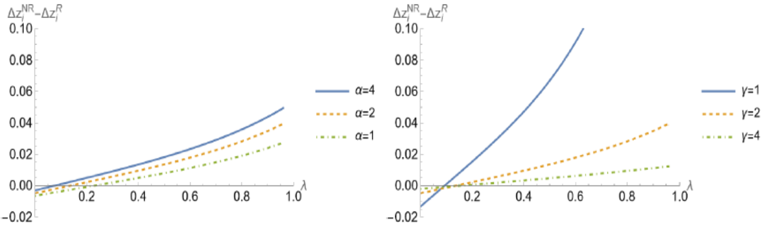
- **Is regulation more effective with sim. or seq. investments?**
- Consider $\Delta z_i^{NR} - \Delta z_i^{R}$, or after rearranging,

$$\left(z_i^{R,Seq} - z_i^{NR,Seq}\right) - \left(z_i^{R} - z_i^{NR}\right).$$

- If positive, regulation would provide firms with stronger incentives to increase z_i under seq. than sim. investments.
- If negative, the opposite ranking applies.

Sequential investments - Regulation

- **Is regulation more effective with sim. or seq. investments?**
- Positive differential, meaning regulation is more effective with seq. investments.
- Strategic uncertainty attenuates the feedback effect.



(a) Differential $\Delta z_i^{NR} - \Delta z_i^R$ at different values of α.

(b) Differential $\Delta z_i^{NR} - \Delta z_i^R$ at different values of γ.

Figure 11: Differentials $\Delta z_i^{NR} - \Delta z_i^R$ over a range of λ.

Discussion

Discussion

- **Three externalities**

1. In the absence of reg. and economies of scope.
   - An increase in k_i makes firm i more competitive.
   - #1: Negative externality on its rivals.

Still without reg., but allow introduce economies of scope.

2. An increase in abatement z_i helps firm i to lower its R&D costs.

- The firm can, then, invest more in k_i.
- #2: Emphasizing the above negative externality.

Discussion

- **Three externalities**

3. Now introduce regulation.
   - An increase in z_i lowers the stringency of t.
   - #3: Positive externality on its rivals.

- The literature only considers #3,

Discussion

- **Three externalities**

3. Now introduce regulation.
    - An increase in z_i lowers the stringency of t.
    - #3: Positive externality on its rivals.

- The literature only considers #3,
    - but the presence of #1 and #2 can lead regulators to...

Discussion

- **Three externalities**

3. Now introduce regulation.
   - An increase in z_i lowers the stringency of t.
   - #3: Positive externality on its rivals.

- The literature only considers #3,
   - but the presence of #1 and #2 can lead regulators to...
   - underestimate k_i,

Discussion

- **Three externalities**

3. Now introduce regulation.
    - An increase in z_i lowers the stringency of t.
    - #3: Positive externality on its rivals.

- The literature only considers #3,
    - but the presence of #1 and #2 can lead regulators to...
    - underestimate k_i,
    - anticipate less pollution, and

Discussion

- **Three externalities**

3. Now introduce regulation.
   - An increase in z_i lowers the stringency of t.
   - #3: Positive externality on its rivals.

- The literature only considers #3,
   - but the presence of #1 and #2 can lead regulators to...
   - underestimate k_i,
   - anticipate less pollution, and
   - undertax.

Discussion

- **Undertaxation.**
- It arises even in the absence of economies of scope, $\lambda = 0$.
    - In that setting, #2 does not exist, but #1 still gives rise to higher k_i, output, and pollution than in traditional models.
    - The regulator, then, responds with a fee that is too lax.
- When economies of scope are present, #2 arises.
    - Leading firms to invest more in both, but specially in abatement.
    - This attenuates the magnitude of undertaxation.

Discussion

- *Practitioners:* When is undertaxation the largest?
  - When pollution is severe, cost of R&D or ER&D is low, or when industry is less competitive.
  - Otherwise, ignoring the multiplicity of investments generates small inefficiencies.

Discussion

- **Sequential investments.**
- Under no reg., a positive feedback effect arises under sim. investments.
  - It helps firms lower their abatement costs.
- In a sequential setting, this feedback effect is broken.
  - R&D investments do not lower abatement costs.
  - Investing more in abatement under sim. than seq. ("abatement differential").

Discussion

- Introducing regulation:
  - Attenuates the abatement differential.
  - But this differential is still positive.
  - This implies that emission fee should be more stringent than under simultaneous investments.
  - Emphasizing the undertaxation problem.

Further research

- Firms coordinating their investment decisions.
  - Their k_i's, their z_i's, or both.
  - Helping at internalizing the above three externalities.
- Asymmetric economies of scope across firms.
  - λ_i and λ_j, but it would make results more untractable.
- Allowing for investments to generate spillover effects across firms:
  - R&D investments, $c - k_i - \beta K_{-i}$, where $\beta \geq 0$.
  - ER&D investments, $e_i = q_i - z_i - \beta_E Z_{-i}$, where $\beta_E \geq 0$.

Motivation

- How Are Patent Decisions Affected by Environmental Regulation?

Motivation

- Patents seek to balance the trade-off between:
  - Monopolization during the patent period, and
  - Providing more incentives to invest in R&D, which eventually benefits all firms.
- Debate:
  - Too long or too short patents?

Motivation

- Large body of literature, but it generally assumes that:
  - Production does not generate environmental damages, and
  - Firms face no environmental regulation.
- Let us relax both assumptions:
  - Allowing for EPA to be present/absent.
  - Allowing for the patent office (PO) to ignore/consider pollution.

Motivation

- **Research questions:**
  - How are patent lengths affected by env. regulation?
  - What if the PO ignores/considers pollution?
  - Measure patent inefficiencies in different contexts.

Literature

- Seminal work by Nordhaus (1969), identifying the trade-off.
- Extended along several dimensions:
  - Optimal "height" of the novelty requirement (legal protection); van Dijk (1996) and O'Donoghue et al. (1998).
  - Licensing to other firms; Denicolò (1996) and Gallini and Scotchmer (2002).
- Game-theoretic modeling:
  - Takalo (2001) and Belleflamme and Peitz (2015).
- Few papers allowing for environmental damages:
  - Gerlagh et al. (2014), but assume R&D is in abatement, not cost-reducing (as opposed to patents literature).
  - Langinier and Chaudhuri (2019) assume fixed patent lengths (which we relax) and exogeneous fees.

Outline

- Patents without environmental regulation
- Patents with environmental regulation
- Extensions:
    - Green innovations
    - Pigouvian EPA
    - Uniform emission fees
    - Fixed patent lenghts
- Discussions.

Model

- Consider inverse demand function $p(Q) = 1 - Q$, and common marginal cost $1 > c > 0$.
- R&D cost is $C(x) = \frac{1}{2}\gamma x^2$, where γ denotes R&D efficiency.
- Three settings:
  - No innovation, *NI*:
    - Perfectly competitive market, yielding $\pi^{NI} = 0$.
  - Innovation:
    - Patent period, *P*: The innovator's cost decreases to $c - \alpha$, where $0 \leq \alpha \leq c$.
    - After the patent expires, *C*: The patent becomes public, zero profits.

Model - Time structure

1. *First stage*: The EPA sets emission fees τ.
   - The EPA's welfare function is
   $$CS + PS - Env,$$
   where $Env = bQ^2$ where $0 \leq b < 1/2$, and $b \geq d$.
   - Allows for $b = d$ or $d = 0$ as special cases.
2. *Second stage*: Patent office (PO) observes τ and chooses patent length $T \geq 0$.
   - The PO's welfare function is
   $$CS + PS,$$
3. *Third stage*: The innovator observes the patent length T and τ, and responds with its R&D investment, $x(T, \tau)$.
4. *Fourth stage*: Firms choose their output level (Cournot).

Benchmark - No regulation

No environmental regulation

Benchmark - No regulation

- **Third stage (output decisions):**
  - *Before the patent expires:*
    - The innovator's monopoly output is $q^m = \frac{1-(c-\alpha)}{2}$ with monopoly price $p^m = \frac{1+(c-\alpha)}{2}$.
    - The innovation is "non-radical" if $p^m > c$, or $\alpha < 1 - c$. (Non-extreme cost-reduction effects.) Otherwise, the patent's legal protection is irrelevant.
    - For the innovator to be the only seller, we need $p^P = c - \varepsilon$, which yields $q^P = 1 - c$, and $\pi^P = \alpha(1 - c)$.
  - *After the patent expires:*
    - Every firm enjoys production cost $c - \alpha$, interacting in a perfectly competitive market.
    - $p(Q^C) = c - \alpha$ and zero profits, $\pi^C = 0$.

Benchmark - No regulation

- **Second stage (R&D investment):**
- The innovator anticipates π^P and π^C, and chooses x to solve

$$\max_{x \geq 0} \overbrace{x\Pi_0(T)}^{\text{Innov.}} + \overbrace{(1-x)\pi^N}^{\text{No innov.}} - \frac{1}{2}\gamma x^2$$

where $\Pi_0(T)$ denotes the return of a successful innovation:

$$\begin{aligned}\Pi_0(T) &= \overbrace{\int_0^T e^{-rt}\pi^P dt}^{\text{Before patent exp.}} + \overbrace{\int_T^{+\infty} e^{-rt}\underbrace{\pi^C}_{0} dt}^{\text{After patent exp.}} \\ &= \frac{\alpha(1-c)\left(1-e^{-rT}\right)}{r}\end{aligned}$$

which is increasing in the patent length, T.

Benchmark - No regulation

- **Second stage (R&D investment):**
- **Lemma 1.** *The innovator's equilibrium investment in R&D is*

$$x_0(T) = \frac{\alpha(1-c)\left(1-e^{-rT}\right)}{\gamma r},$$

which is increasing and concave in T, increasing in α, and decreasing in γ and r.

  - Intuitively, longer patents provide stronger incentives to invest, but at a decreasing rate.
  - More incentives to invest if the innovation is less costly (lower γ) or more effective (higher α).

Benchmark - No regulation

- **First stage (Patent length):**
- PO anticipates $x_0(T)$ and solves

$$\max_{T\geq 0} \; x_0(T) \underbrace{\left(\int_0^T e^{-rt} W_0^P dt + \int_T^{+\infty} e^{-rt} W_0^C dt \right)}_{\text{Welfare with innov.}}$$

$$+(1 - x_0(T)) \underbrace{\int_0^{+\infty} e^{-rt} W_0^N dt}_{\text{Welfare without innov.}}$$

$$-\frac{1}{2}\gamma \left[x_0(T)\right]^2$$

where $\pi^P > \pi^C = 0$, but $W_0^P > W_0^C$ iff $d > \frac{\alpha^2}{2[\alpha^2+2(1-c)]}$.

Benchmark - No regulation

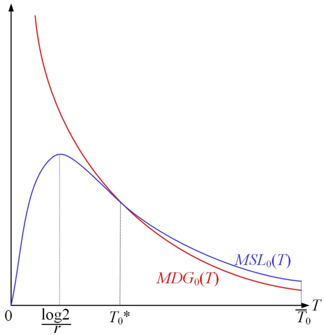
- **First stage (Patent length):**
  - Differentiating the PO's problem with respect to T, yields

$$\underbrace{\frac{\partial x_0(T)}{\partial T} S_0(T)}_{MDG_0(T)} = \underbrace{x_0(T)\left(\overbrace{\gamma \frac{\partial x_0(T)}{\partial T}}^{\text{R\&D cost}} - \overbrace{\frac{\partial S_0(T)}{\partial T}}^{\text{CS loss}}\right)}_{MSL_0(T)}.$$

  - Marginal dynamic gain, $MDG_0(T)$, from a longer patent.
    - The firm increases its R&D intensity, $x_0(T)$, which increases welfare.
  - Marginal static loss, $MSL_0(T)$, from a longer patent.
    - Larger R&D cost (first term) and lower CS from a longer monopoly (second term).

Benchmark - No regulation

- **First stage (Patent length):**
  - $MDG_0(T)$ and $MSL_0(T)$ curves.



Benchmark - No regulation

- **First stage (Patent length):**
- **Proposition 1.** *When the EPA is absent, the optimal patent length is*

$$T_0(d) = \frac{1}{r} \log \left[\frac{2[1-(c-\alpha)]}{\alpha} \right],$$

which decreases in α and c.

$$x_o = \frac{\alpha(1-c)[\alpha + 2(1-c)]}{2\gamma r[1-(c-\alpha)]}$$

Introducing environmental regulation

Introducing environmental regulation

- In this context, equilibrium behavior in stages 2-4 is analogous to that in stages 1-3 above.
- But increasing firms'costs in all scenarios: from c to $c + \tau$ in the absence of innovation;
- and from $c - \alpha$ to $c - \alpha + \tau$ when innovation takes place.

Introducing environmental regulation

- **Third stage: R&D investment.**

$$x_0(T,\tau) = \frac{\alpha(1-c-\tau)\left(1-e^{-rT}\right)}{\gamma r}$$

- Which is decreasing in the fee and $0 < x_0(T,\tau) < x_0(T)$;
  - $0 < x_0'(T,\tau) < x_0'(T)$

Introducing environmental regulation

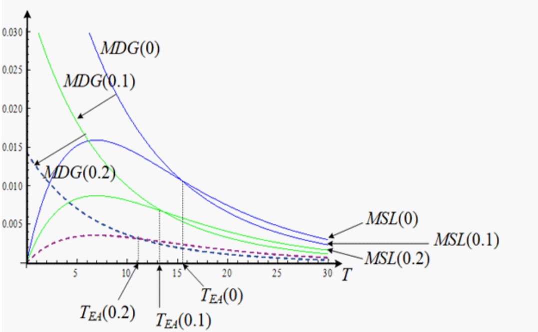
- **Second stage: Patent**

$$T_0\left(d\right) = \frac{1}{r}\log\left[\frac{2\left[1-\left(c-\alpha\right)+\tau\right]}{\alpha}\right].$$

- PO sets shorter patents when firms' initial costs are higher (c or τ)
  - the patent gives rise to a smaller cost-reduction effect (for a given value of α)
  - reducing the social benefit of the patent.

Introducing environmental regulation

- **Second stage: Patent length**



Introducing environmental regulation

- **First stage: Emission fee**

$$\max_{\tau\geq 0}\; x(T,\tau)\underbrace{\left(\int_0^T e^{-rt}W_{EA}^P dt+\int_T^{+\infty} e^{-rt}W_{EA}^C dt\right)}_{\text{Welfare with innov.}}$$

$$+(1-x(T,\tau))\underbrace{\int_0^{+\infty} e^{-rt}W_{EA}^N dt}_{\text{Welfare without innov.}}$$

$$-\frac{1}{2}\gamma\left[x\left(T,\tau\right)\right]^2$$

Introducing environmental regulation

- **First stage: Emission fee**
- The EA, then, faces a tradeoff
  - a more stringent fee reduces expected pollution if an innovation occurs, yielding a welfare gain;
  - but it also reduces firms'investment, making innovation less likely.
- No innovation, $\tau^N = \frac{2d(1-c)}{1+2d}$
- Innovation, linear combination between $\tau^P = \frac{2d(1-c)-\alpha}{1+2d}$ and $\tau^C = \frac{2d(1-c)+2d\alpha}{1+2d}$.
  - $\tau^C > \tau^P > \tau^N$.

Introducing environmental regulation

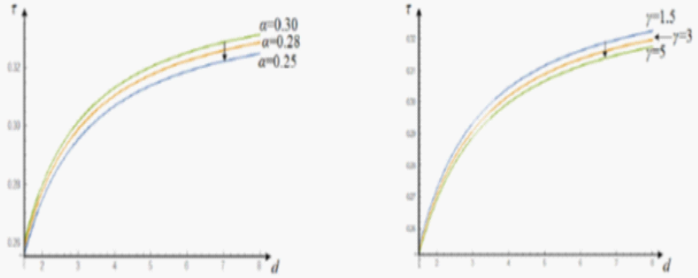


Fig 3a. Fee τ^*, changes in α.

Fig 3b. Fee τ^*, changes in γ.

Introducing environmental regulation

Comparison $\Delta T \equiv T_o - T_{EA}$:

		Regulation			No regulation		Comparison	
		τ^*	T_{EA}	x_{EA}	T_0	x_0	ΔT	Δx
	Benchmark	0.2555	9.6409	0.0802	15.4045	0.4365	5.7636	0.3563
Higher α	$\alpha = 0.18$	0.2517	10.6703	0.0642	17.4112	0.3299	6.7409	0.2657
	$\alpha = 0.20$	0.2526	10.3235	0.0693	16.7398	0.3611	6.4163	0.2918
	$\alpha = 0.22$	0.2536	10.024	0.0740	16.1548	0.3917	6.1308	0.3177
	$\alpha = 0.24$	0.2548	9.7612	0.0783	15.6398	0.4217	5.8786	0.3434
	$\alpha = 0.26$	0.2563	9.5271	0.0820	15.1822	0.4512	5.6551	0.3692
	$\alpha = 0.28$	0.2579	9.3161	0.0853	14.7727	0.4802	5.4566	0.3949
	$\alpha = 0.30$	0.2598	9.1239	0.0880	14.4036	0.5088	5.2797	0.4208
	$\alpha = 0.32$	0.2619	8.9473	0.0901	14.0691	0.5370	5.1218	0.4469
Higher c	$c = 0.35$	0.4900	11.8774	0.1853	19.7408	0.9329	7.8634	0.7476
	$c = 0.40$	0.4531	11.5529	0.1677	19.1692	0.8529	7.6163	0.6852
	$c = 0.45$	0.4162	11.2186	0.1504	18.5630	0.7734	7.3444	0.6230
	$c = 0.50$	0.3792	10.8738	0.1335	17.9176	0.6944	7.0438	0.5609
	$c = 0.55$	0.3422	10.5180	0.1170	17.2277	0.6161	6.7097	0.4991
	$c = 0.60$	0.3051	10.1504	0.1009	16.4866	0.5385	6.3362	0.4376
	$c = 0.65$	0.2679	9.7705	0.0853	15.6862	0.4618	5.9157	0.3765
	$c = 0.70$	0.2307	9.3774	0.0703	14.8160	0.3864	5.4386	5.4386

Conclusions

- Overall, patents are shorter with than without environmental policy when the EA is less flexible than the PO, but become longer otherwise.
- Hence, considering the EA's administrative ability to revise emission fees is important, since this flexibility can produce different effects in the PO's patent lengths decisions.
- Ignoring the presence of environmental policy when designing patent lengths will induce unnecessarily long monopolies, ultimately reducing social welfare.