Intermediate Microeconomic Theory

Tools and Step-by-Step Examples

Chapter 10: Monopoly

Important Dates

- **Midterm #2**.
  - Take Home – You will receive it on Thursday March 20th
  - Due Date: Tuesday March 25th
  - It will cover:
    - Measuring Welfare Changes (CH 5)
    - Choice under Uncertainty (CH 6)
    - Production Functions (CH 7)
    - Cost Minimization (CH 8)
    - Monopoly (CH 10)

  - Recitation Friday March 21st.

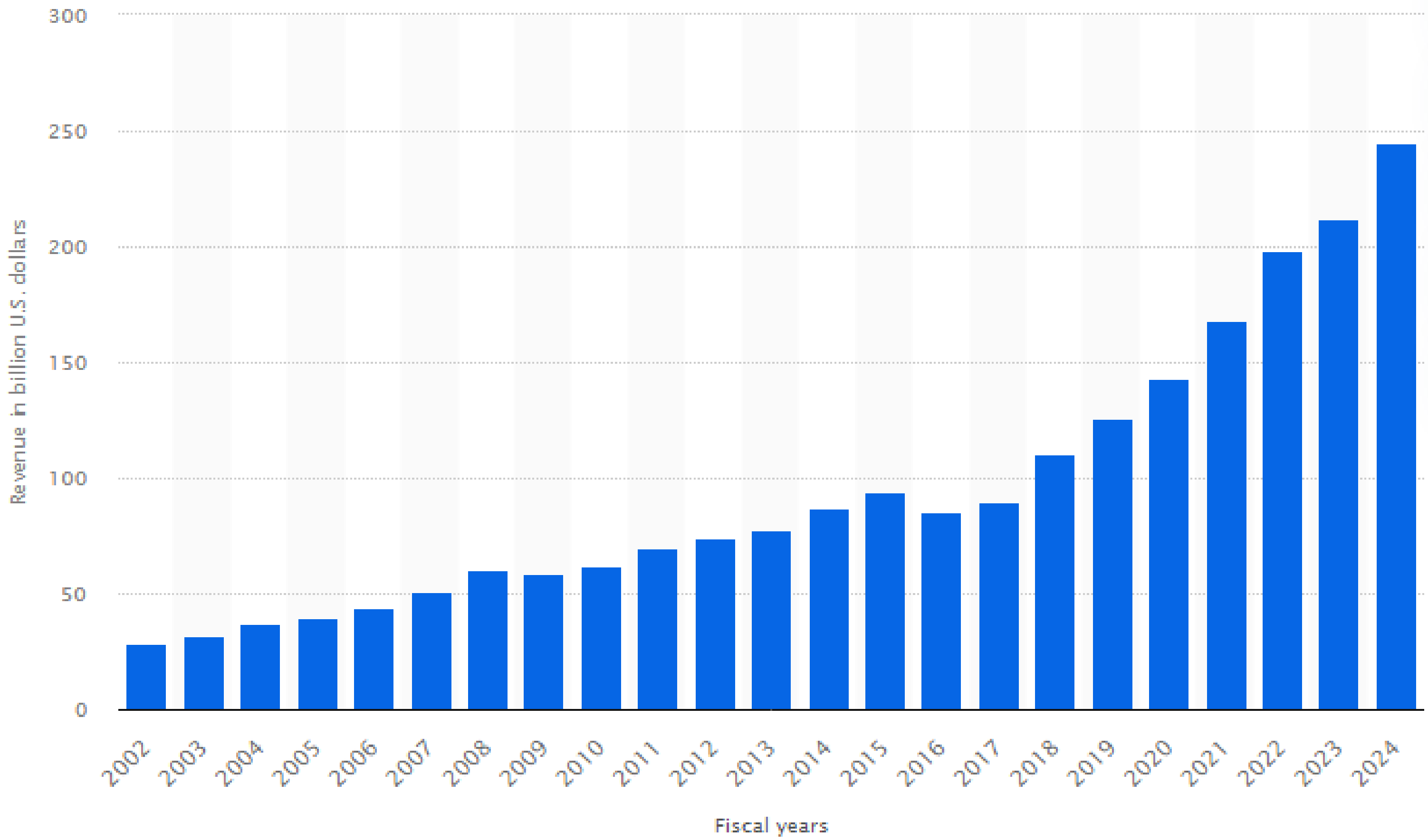
Outline

- Barriers to entry
- Profit Maximization Problem (PMP)
- Common Misunderstandings of Monopoly
- The Lernex Index and Inverse Elasticity Pricing Rule
- Multiplant Monopolist
- Welfare Analysis under Monopoly
- Advertising in Monopoly
- Monopsony

- Microsoft Windows version 3.0 in 1990 cemented Microsoft's position as a software monopoly.
- Soon after Windows 3.0 was released, Microsoft released Excel 3.0 for Windows and Word for Windows 2.0.
- These products received great reviews and became the top sellers of their categories. (New York Times, 11/5/95)

Revenue of Microsoft from 2002 to 2024

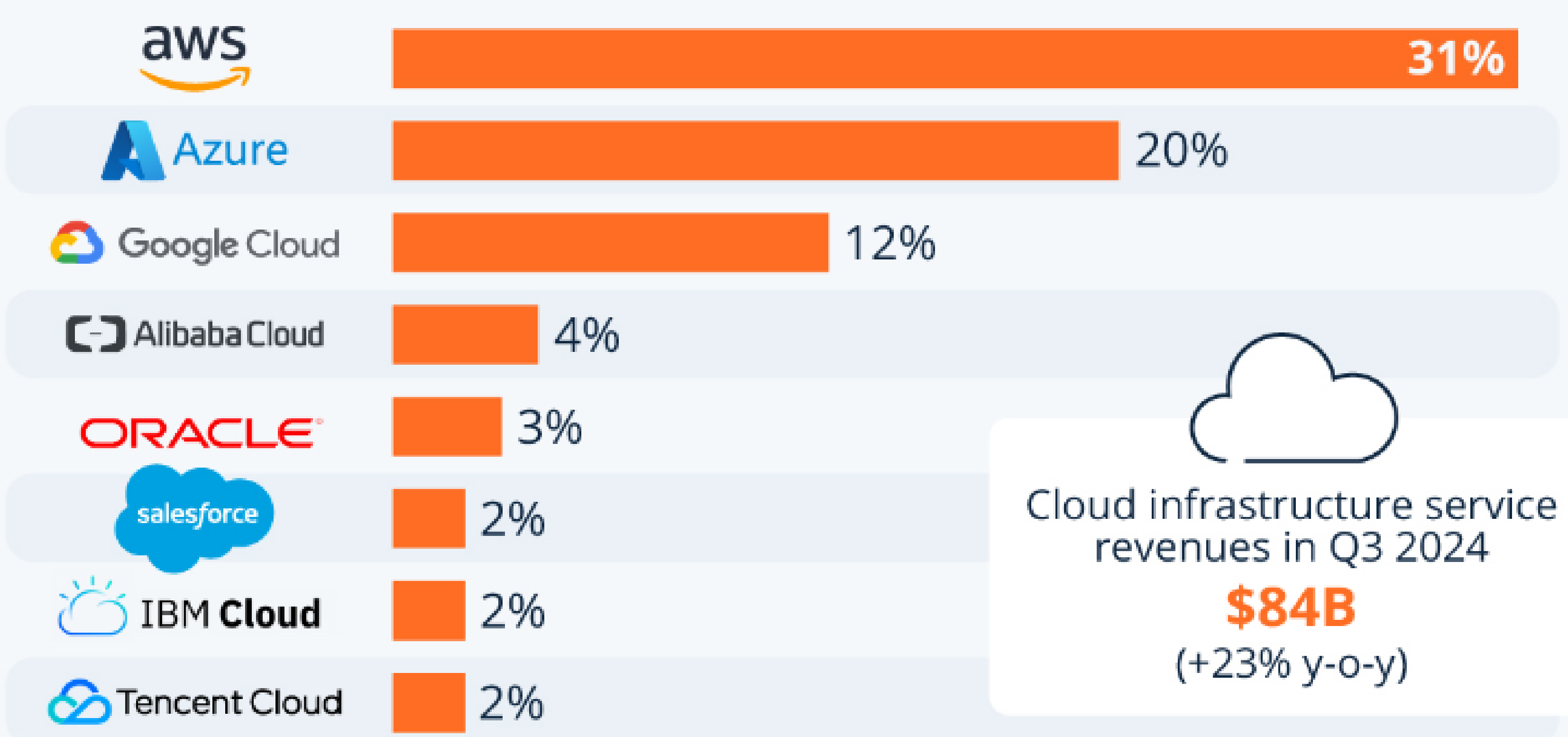
(in billion U.S. dollars)



Details: Worldwide; United States; 2002 to 2024

Amazon Maintains Dominant Lead in the Cloud Market

Worldwide market share of leading cloud infrastructure service providers in Q3 2024*



Barriers to Entry

Barriers to entry

"Why do monopolies exist in the first place if they are bad for society?"

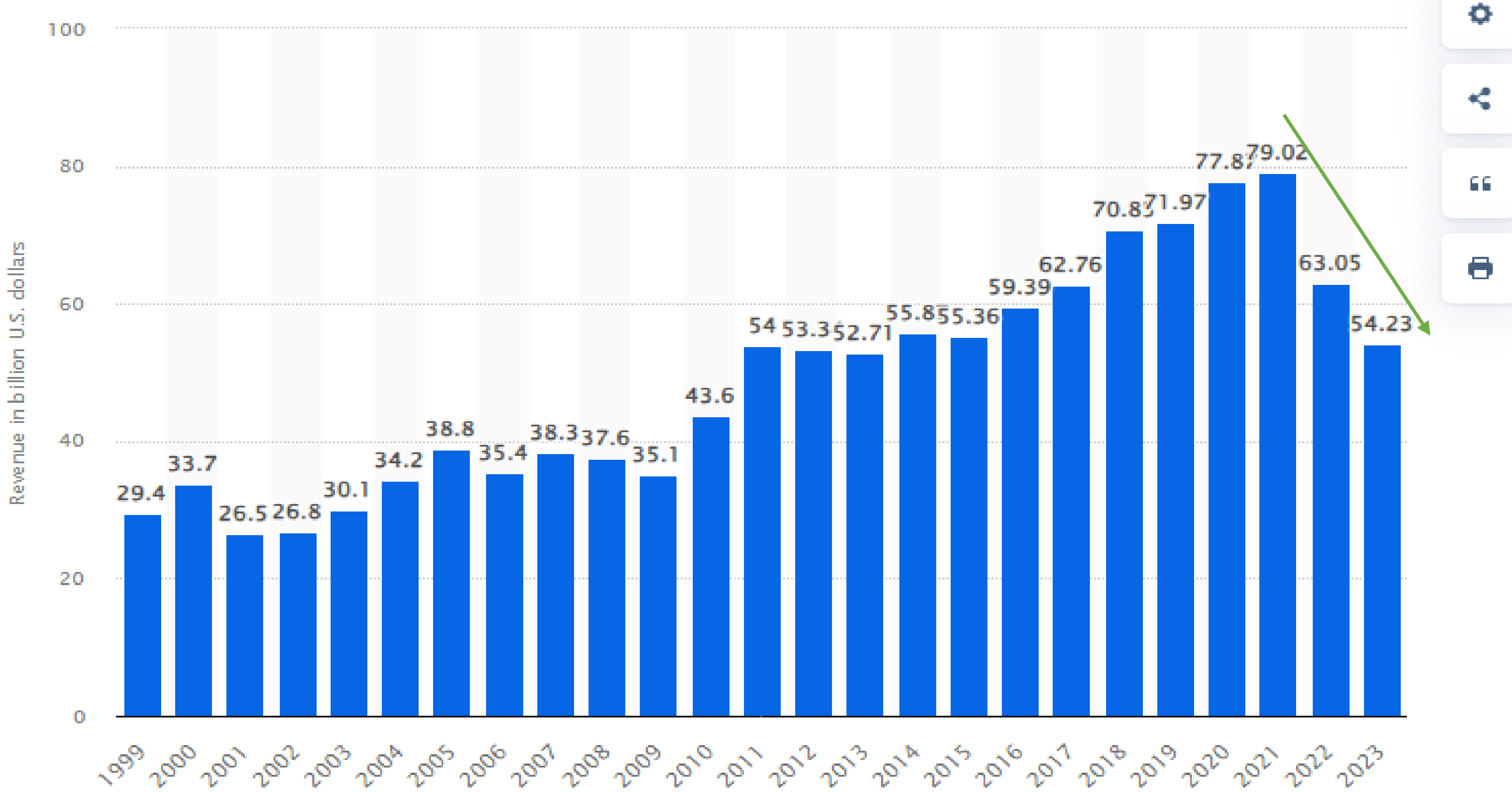
- Structural barriers: Incumbent firms may have advantages that are unattractive for potential entrants.
  - Cost advantage (e.g., superior technology)
  - Demand advantage (e.g., large group of loyal customers)
- Legal barriers: Incumbents firms may be legally protected.
  - *Example*: Patents.
- Strategic barriers: Incumbent firms can take actions to deter entry, by building a reputation of being a tough competitor.
  - *Example*: Price wars.

TECHNOLOGICAL SUPERIORITY

- A firm that maintains a consistent technological advantage over potential competitors can establish itself as a monopolist.
- Example: Intel was technologically superior over other firms from the 1970s to the 1990s.
- Technological superiority is typically not a barrier to entry over the longer term.

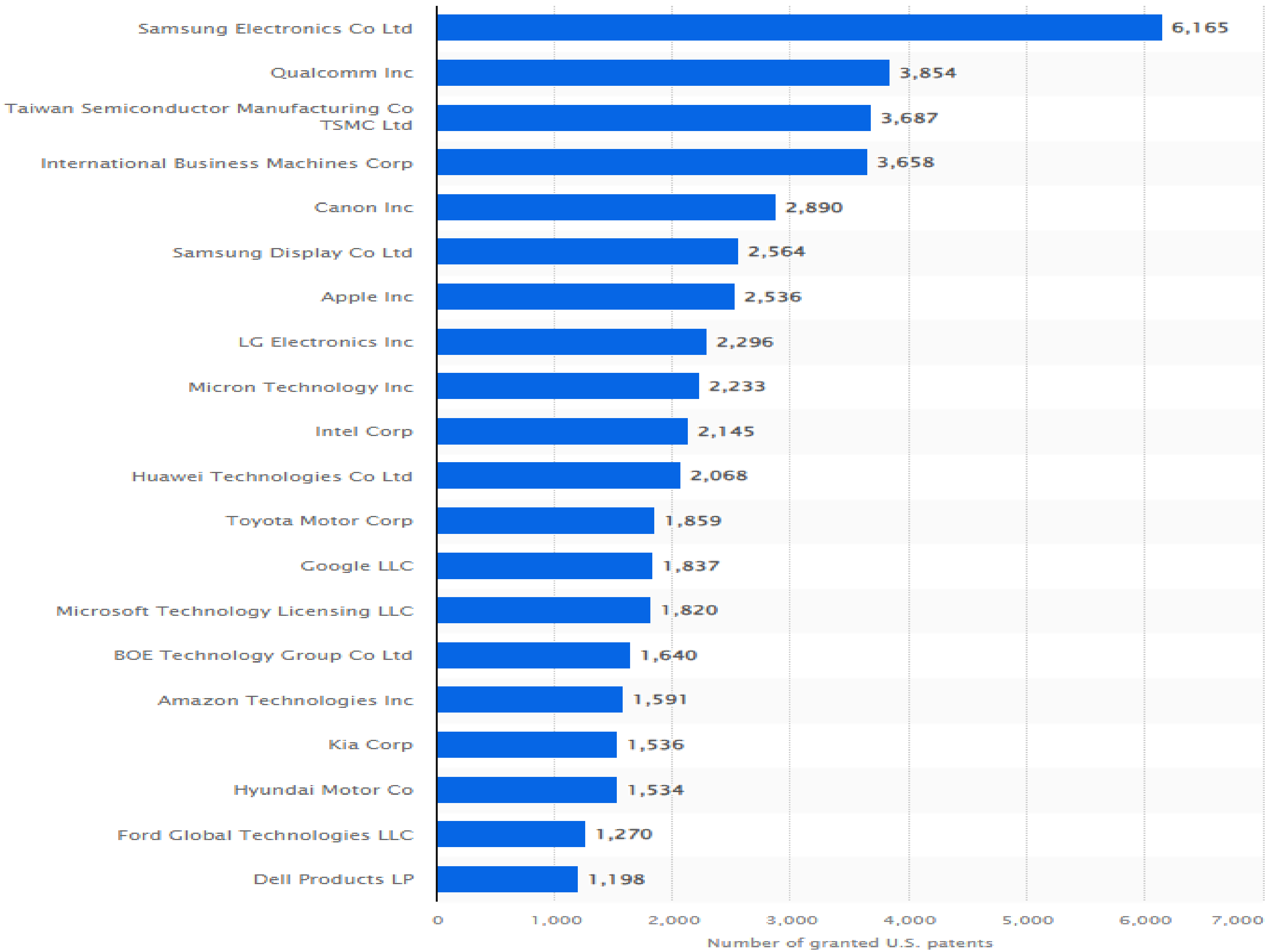
Revenue of Intel from 1999 to 2023

(in billion U.S. dollars)



Details: Worldwide; Intel; 1999 to 2023

Companies with the most U.S patents granted in 2023



Profit Maximization Problem (PMP)

Profit Maximization Problem

- In a monopolized industry,
  - A single firm decides the output level, $q = Q$.
  - A change in q affects market prices, as measured by the inverse demand function $p(q)$, which decreases in q.
  - *Example* (linear inverse demand):

$$p(q) = a - bq, \text{ where } a, b > 0$$

    - When the monopolist sells few units (low values of q), consumers are willing to pay a relatively high price for the scarce good.
    - As the firm offers more units (larger values of q), consumers are willing to pay less for the relatively abundant good.

Profit Maximization Problem

- PMP: The monopolist chooses its output q to maximize its profits π

$$\max_{q} \ \pi = TR(q) - TC(q) = p(q)q - TC(q).$$

- Differentiating with respect to q,

$$p(q) + \frac{\partial p(q)}{\partial q} q - \frac{\partial TC(q)}{\partial q} = 0.$$

- Rearranging,

$$\underbrace{p(q) + \frac{\partial p(q)}{\partial q} q}_{\text{Marginal revenue, } MR(q)} = \underbrace{\frac{\partial TC(q)}{\partial q}}_{\text{Marginal cost, } MC(q)}.$$

Profit Maximization Problem

- Therefore, to maximize profits, the monopolist increases its output q until

$$MR(q) = MC(q).$$

  - If $MR(q) > MC(q)$, the monopolist would have incentives to increase output q because its revenues increases more than its cost.
  - If $MR(q) < MC(q)$, the monopolist would have incentives to decrease its output q.

Profit Maximization Problem

- A closer look at marginal revenue,

$$MR(q) = \underbrace{p(q)}_{\text{Positive effect}} + \underbrace{\frac{\partial p(q)}{\partial q} q}_{\text{Negative effect}}.$$

- When monopolist increases output by 1 unit, this additional unit produces 2 effects on firm's revenue:
  - Positive effect. If the firm sells 1 more unit, it earns $p(q)$, and the firm's revenue increases.
  - Negative effect. When offering 1 more unit, the firm needs to decrease the price of previous units sold, $\frac{\partial p(q)}{\partial q} < 0$.
- In summary, the total effect of increasing output must exactly offset the additional costs of producing 1 more unit, $MR(q) = MC(q)$.

Profit Maximization Problem

- *Example 10.1: Positive and negative effects of selling more units.*
  - Consider $p(q) = 10 - 3q$. If the firm were to marginally increase its output,

$$\begin{aligned} MR(q) &= p(q) + \frac{\partial p(q)}{\partial q} q \\ &= (10 - 3q) + (-3)q \\ &= 10 - 6q. \end{aligned}$$

  - If the firm sells $q = 2$ units,

$$TR(1) = p(2)2 = (10 - 3 \times 2)2 = \$8.$$

Profit Maximization Problem

- *Example 10.1* (continued):
  - Evaluating $MR(q)$ at $q = 2$ units yields

$$MR(2) = (10 - 3 \times 2) + (-3)2 = 4 - 6 = -\$2.$$

  - The monopolist's revenue experiences:
    - A positive effect of $\$4$ because it now sells 1 more unit at price $\$4$.
    - A negative effect because selling 1 more unit entails applying a price discount of $\$3$ on all previous units.
    - Overall, these two effect generates a total (net) decrease in revenue of $\$2$.

Profit Maximization Problem

- *Example 10.2: Finding marginal revenue with linear demand.*
  - Consider $p(q) = a - bq$. Marginal revenue is

$$MR(q) = p(q) + \frac{\partial p(q)}{\partial q} q = (a - bq) + (-b)q = a - 2bq.$$

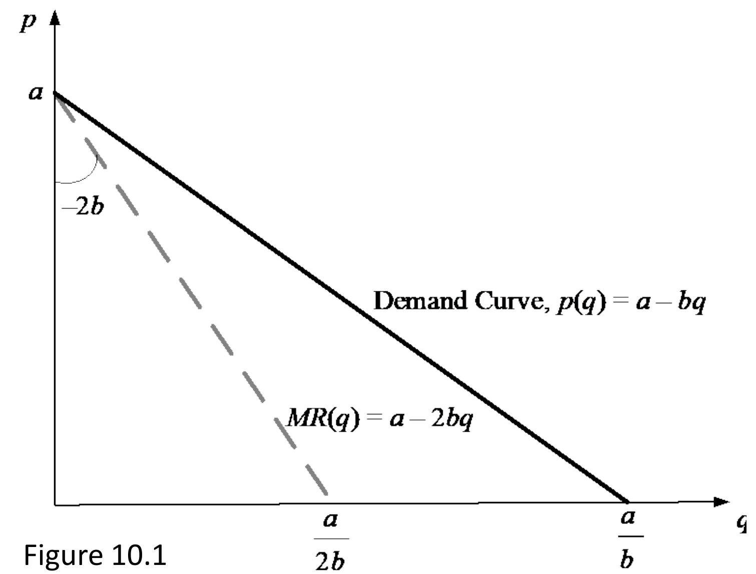


Figure 10.1

Profit Maximization Problem

- Two properties of marginal revenue curve:

  1) *$MR(q)$ lies below the demand curve.*
     We need,

$$MR(q) \leq p(q),$$

$$p(q) + \frac{\partial p(q)}{\partial q} q \leq p(q) \rightarrow \frac{\partial p(q)}{\partial q} q \leq 0.$$

  2) *$MR(q)$ and the demand curve originate at the same height.*
     At $q = 0$,

$$p(0) = a - b \times 0 = a,$$

$$MR(0) = p(0) + \frac{\partial p(q)}{\partial q} q = p(0) = a.$$

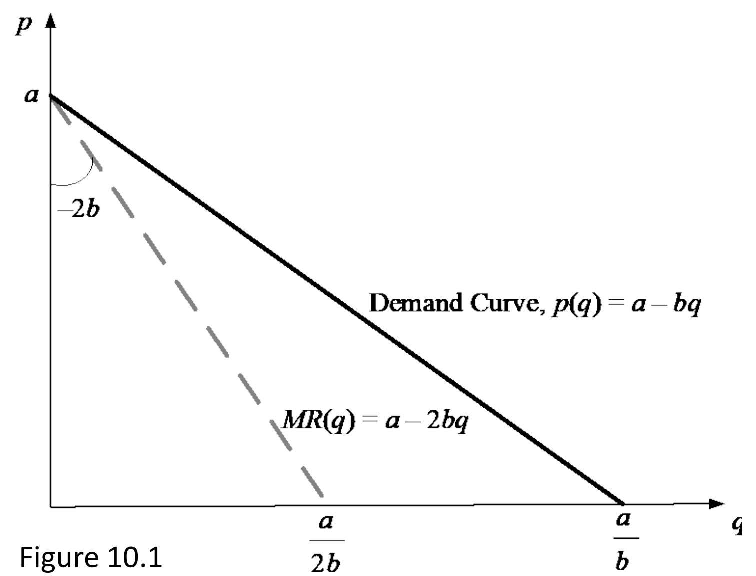


Figure 10.1

Profit Maximization Problem

- *Example 10.3: Finding monopoly output with linear demand.*
  - Consider $p(q) = a - bq$, and $TC(q) = cq$, where $c > 0$.
  - The monopolist maximizes its profits by solving

$$\max_q \pi = TR(q) - TC(q) = \underbrace{(a - bq)q}_{TR} - \underbrace{cq}_{TC}.$$

  - Differentiating with respect to q yields

$$a - 2bq - c = 0.$$

  - Rearranging,

$$\underbrace{a - 2bq}_{MR(q)} = \underbrace{c}_{MC(q)}.$$

Profit Maximization Problem

- *Example 10.3* (continued):
  - Rearranging,

$$a - c = 2bq.$$

  - Solving for output q,

$$q^M = \frac{a-c}{2b}.$$

  - We find the monopoly price by inserting this output into the inverse demand function

$$p(q^M) = a - bq^M = a - b\overbrace{\left(\frac{a-c}{2b}\right)}^{q^M}$$

$$= \frac{2ab - b(a-c)}{2b} = \frac{a+c}{2}.$$

Profit Maximization Problem

- *Example 10.3* (continued):

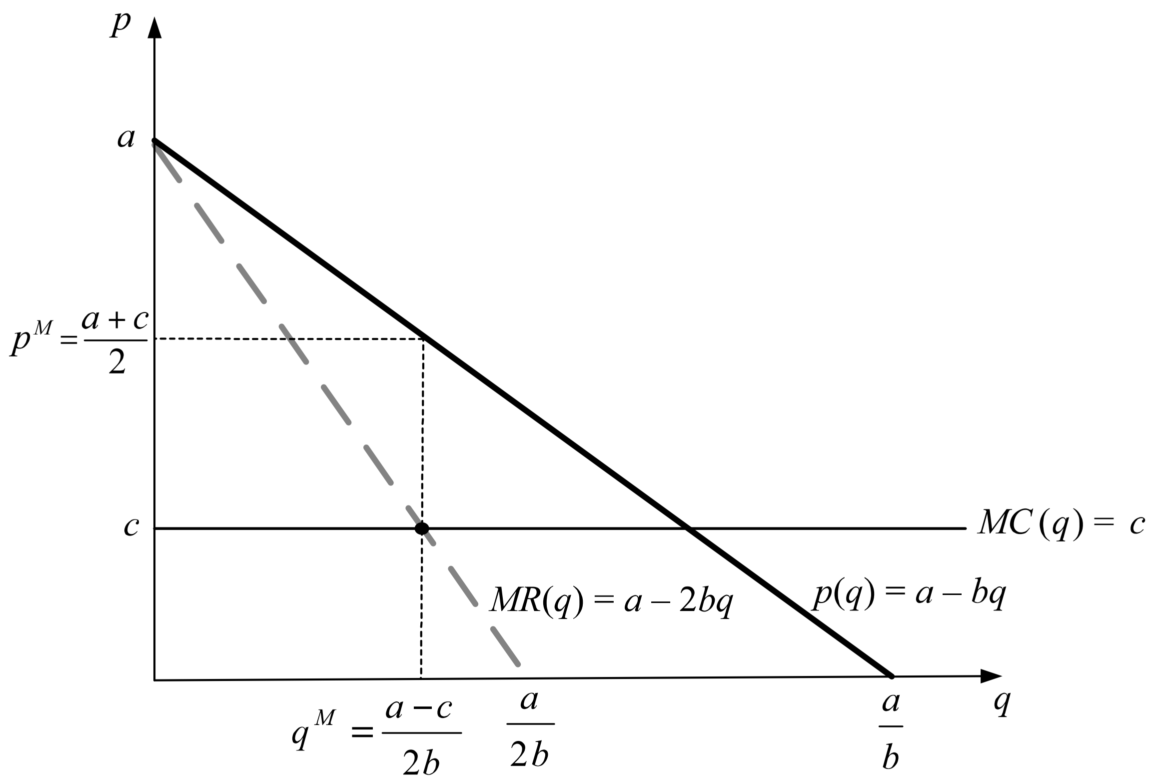


Figure 10.2

Profit Maximization Problem

- *Example 10.3* (continued):
  - Monopoly profits are

$$
\begin{aligned}
\pi^M &= p(q^M)q^M - cq^M \\
&= \frac{a+c}{2} \cdot \frac{a-c}{2b} - c\,\frac{a-c}{2b} \\
&= \left(\frac{a+c}{2} - c\right)\frac{a-c}{2b} \\
&= \frac{(a-c)^2}{4b}.
\end{aligned}
$$

Profit Maximization Problem

- *Example 10.3* (continued):
  - Consumer surplus under this monopoly is

$$CS^M = \frac{1}{2}\underbrace{\left(a - \frac{a+c}{2}\right)}_{\text{height}} \cdot \underbrace{\frac{a-c}{2b}}_{\text{base}} = \frac{1}{2} \cdot \frac{2a-a-c}{2} \cdot \frac{a-c}{2b} = \frac{(a-c)^2}{8b}.$$

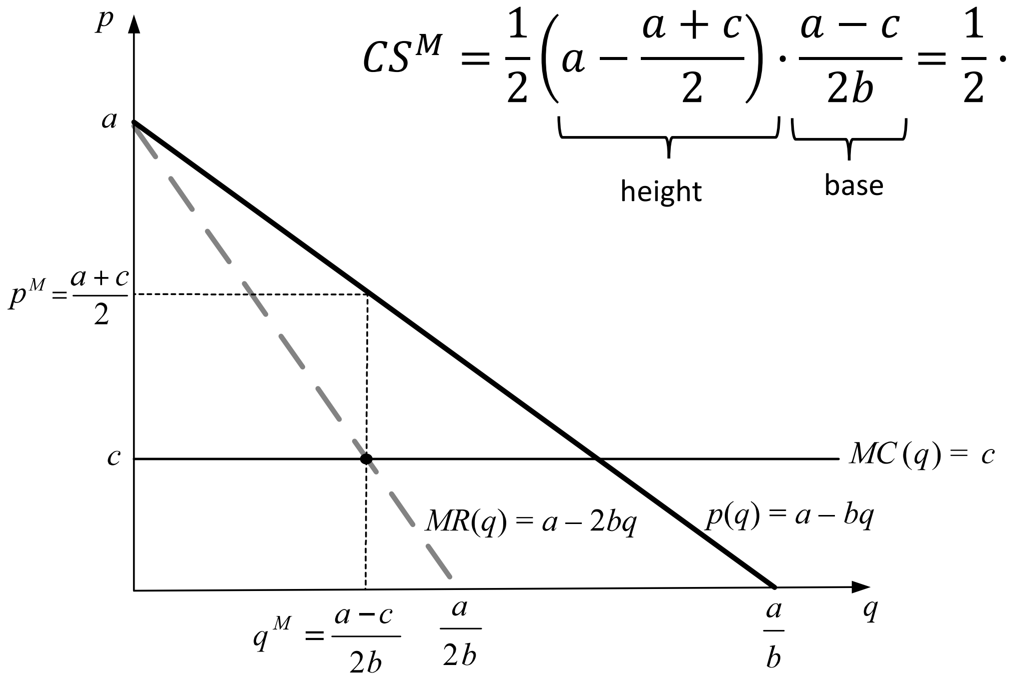


Figure 10.2

Profit Maximization Problem

- *Example 10.3* (continued):
  - If inverse demand is $p(q) = 10 - q$ (i.e., $a = 10$ and $b = 1$), and $TC(q) = 4q$ (i.e., $c = 4$)
    - $q^M = \frac{a-c}{2b} = \frac{10-4}{2} = 3$ units.
    - $p^M = a - bq^M = \$7.$
    - $\pi^M = \frac{(a-c)^2}{4b} = \frac{(10-4)^2}{4} = \frac{36}{4} = \$9.$
    - $CS^M = \frac{(a-c)^2}{8b} = \frac{(10-4)^2}{8} = \frac{36}{8} = \$4.5.$

Common misunderstandings of Monopoly

Common Misunderstandings

1. *Monopolies do not set infinitely high prices.*
   - While the monopolist is the only firm in its industry, it faces a demand curve $p(q)$, such as $p(q) = a - bq$.
   - Setting higher prices might be attractive but could lead to fewer sales.
   - This trade-off implies the monopolist does not set an infinitely high price because it would imply no sales at all.
     - In example 10.3, any price above $p = \$a$ (e.g., $\$10$ if $a = 10$) entails no sales.

Thanksgiving 2024 – Traveling from Pullman-Seattle

Round trip · 1 · Economy

Pullman PUW → Seattle SEA | Fri, Nov 22 – Sat, Nov 30

All filters · Stops · Airlines · Bags · Price · Times · Emissions · Connecting airports · Duration

Best — Based on price and convenience

Cheapest Starts from $856 — Additional options based primarily on price

Top departing flights

Ranked based on price and convenience · Prices include required taxes + fees for 1 adult. Optional charges and bag fees may apply. Passenger assistance info. Sort by:

5:00 AM – 6:13 AM Alaska · American · Operated by Horizon Air as Al…	1 hr 13 min PUW–SEA	Nonstop	83 kg CO2e Avg emissions	$1,007 round trip
12:54 PM – 2:06 PM Alaska · Operated by Horizon Air as Alaska Horizon	1 hr 12 min PUW–SEA	Nonstop	83 kg CO2e Avg emissions	$1,007 round trip
6:59 PM – 8:12 PM Alaska · Operated by Horizon Air as Alaska Horizon	1 hr 13 min PUW–SEA	Nonstop	83 kg CO2e Avg emissions	$1,007 round trip

Thanksgiving 2024 – Traveling from Spokane-Seattle

Round trip · 1 · Economy

Spokane GEG ⇄ Seattle SEA | Fri, Nov 22 | Sat, Nov 30

All filters | Stops | Airlines | Bags | Price | Times | Emissions | Connecting airports | Duration

Best
Based on price and convenience

Cheapest Starts from $187
Additional options based primarily on price

Top departing options

Ranked based on price and convenience · Prices include required taxes + fees for 1 adult. Optional charges and bag fees may apply. Passenger assistance info.

Sort by:

6:15 AM – 7:40 AM Delta	1 hr 25 min GEG–SEA	Nonstop	70 kg CO2e -17% emissions	$187 round trip
1:40 PM – 3:00 PM Delta · Operated by SkyWest DBA Delta Connection	1 hr 20 min GEG–SEA	Nonstop	84 kg CO2e Avg emissions	$187 round trip
5:00 AM – 6:28 AM Alaska	1 hr 28 min GEG–SEA	Nonstop	53 kg CO2e -37% emissions	$212 round trip

Common Misunderstandings

2. The monopolist does not have a supply curve.

- A common misunderstanding is to consider that q^M, where $MR(q) = MC(q)$, constitutes the monopolist's supply curve.
- In perfectly competitive markets, the firm observes the given market price offers the output that satisfies $p = MC(q)$, obtaining the supply function $q(p)$.
- In a monopoly, the monopolist determines output and price simultaneously.
    - In example 10.3, when the monopolist chooses $q^M = 3$ units, it simultaneously determines $p^M = 10 - 3 = \$7$, not allowing the firm to choose different output levels for a given market price of $p^M = \$7$.

Common Misunderstandings

3. *The monopolist produces in the elastic portion of the demand curve.*
    - Goods with few (or no) close substitutes tend to have a relatively inelastic demand curve.
    - Monopolies often produce goods with no close substitutes. However, it is does not mean that it produces in the inelastic portion of the demand curve.

Common Misunderstandings

3. *The monopolist produces in the elastic portion of the demand curve.*

- Consider the formula of price elasticity of demand

$$\varepsilon_{q,p} = \frac{\%\Delta q}{\%\Delta p},$$

  - If the monopolist produces in the inelastic portion of the demand curve, $|\varepsilon_{q,p}| < 1$, an increase in price by 1% reduces sales by less than 1%. It would increase its price, as sales would not be greatly affected but it would not be profit maximizing.
  - If it produces in the elastic segment, $|\varepsilon_{q,p}| > 1$, an increase in price by 1% reduces sales by more than 1%. The firm does not have incentives to adjust its price.

Common Misunderstandings

- *Example 10.5*: *Price elasticity of output q^M under linear demand.*
  - Consider the monopolist in example 10.3, facing $p(q) = 10 - q$.
  - We found $q^M = 3$ units, and $p^M = \$7$.
  - We find price elasticity as

$$\varepsilon_{q,p} = \frac{\%\Delta q}{\%\Delta p} = \frac{\Delta q}{\Delta p} \cdot \frac{p}{q}.$$

  - If the change in price is small, $\varepsilon_{q,p} = \frac{\partial q(p)}{\partial p} \cdot \frac{p}{q}$.

Common Misunderstandings

- *Example 10.5* (continued):
  - From the inverse demand function, we obtain the direct demand function, $q(p) = 10 - p$. Then,

$$\varepsilon_{q,p} = \frac{\partial q(p)}{\partial p} \cdot \frac{p^M}{q^M} = -1\frac{7}{3} \cong -2.33.$$

  - If the monopolist increases prices by 1%, its sales decrease by 2.33%.
  - Therefore, $\left|\varepsilon_{q,p}\right| = 2.33 > 1$ → the monopolist sets a price p^M lying in the elastic portion of the demand curve.

The Lernex Index and Inverse Elasticity Pricing Rule

The Lernex Index

- We can rewrite the profit-maximizing condition for the monopolist, $MR(q) = MC(q)$, to show a relationship between margin, $p - MC(q)$, and price elasticity, $\varepsilon_{q,p}$,

$$p(q) + \frac{\partial p(q)}{\partial q} q = MC(q).$$

- Marginal revenue can be rearranged as

$$MR(q) = p\left(1 + \frac{\partial p(q)}{\partial q} \cdot \frac{q}{p}\right),$$

$$MR(q) = p\left(1 + \frac{1}{\frac{\partial q(p)}{\partial p} \cdot \frac{p}{q}}\right) = p\left(1 + \frac{1}{\varepsilon_{q,p}}\right).$$

The Lernex Index

- Substituting this expression of $MR(q)$ into $MR(q) = MC(q)$,

$$p\left(1 + \frac{1}{\varepsilon_{q,p}}\right) = MC(q).$$

- Rearranging, $p + p\frac{1}{\varepsilon_{q,p}} = MC(q)$, or $p - MC(q) = -p\frac{1}{\varepsilon_{q,p}}$.
- Dividing both sides by p yields,

$$\frac{p - MC(q)}{p} = -\frac{1}{\varepsilon_{q,p}}.$$

- Which is known as the "Lernex Index":
  - A monopolist's ability to set a price above marginal cost is inversely related to the price elasticity of demand.

The Lernex Index

- The “Lernex Index” is also known as the “markup index” because it measures the price markup over marginal cost.
  - As demand becomes relatively elastic, (i.e., a more negative number) the price markup decreases.
    - *Example*: If $\varepsilon_{q,p} = -4$,

$$-\frac{1}{\varepsilon_{q,p}} = -\frac{1}{-4} = 0.25,$$

price markup over marginal cost decreases to 25%.

The Lernex Index

- As demand becomes relatively inelastic, the price markup increases.
  - *Example*: If $\varepsilon_{q,p} = -0.5$,

$$-\frac{1}{\varepsilon_{q,p}} = -\frac{1}{-0.5} = 2,$$

price markup of 200%.

The Lernex Index

- *Example 10.6*: *Lernex index with a linear demand*.
  - Consider market inverse demand function is $p(q) = 10 - q$.
  - Solving for q, we obtain direct demand $q(p) = 10 - p$.
  - Which yields and elasticity of

$$\varepsilon_{q,p} = \frac{\partial q(p)}{\partial p} \cdot \frac{p}{q}$$
$$= -1 \frac{p}{10 - p}.$$

The Lernex Index

- *Example 10.6* (continued):
  - Assuming $MC(q) = 4$, the Lernex Index becomes

$$\frac{p - MC(q)}{p} = -\frac{1}{\varepsilon_{q,p}},$$

$$\frac{p-4}{p} = -\left(\frac{1}{-1\,\frac{p}{10-p}}\right).$$

The Lernex Index

- *Example 10.6* (continued):
  - Rearranging terms,
$$\frac{p-4}{p} = \frac{10-p}{p}.$$
  - Which simplifies to
$$p - 4 = 10 - p.$$
  - And solving for price, $p = \$7$.

The Lernex Index

- *Example 10.7*: *Lernex index with constant elasticity demand*.
  - Consider monopolist facing demand curve
$$q(p) = 5p^{-\varepsilon}.$$
  - Assuming $MC(q) = \$4$, the Lernex Index becomes
$$\frac{p-4}{p} = -\frac{1}{\varepsilon_{q,p}}.$$

The Lernex Index

- *Example 10.7* (continued):
  - If demand curve is $q(p) = 5p^{-2}$ (i.e., $\varepsilon = -2$),

$$\frac{p-4}{p} = -\frac{1}{-2},$$

    which simplifies to $2p - 8 = p$, or $p = \$8$.

  - If demand function changes to $q(p) = 5p^{-5}$,

$$p = \frac{20}{4} = \$5.$$

    As demand becomes more elastic, price decreases.

Inverse elasticity pricing rule (IEPR)

- Using the Lernex index, and solving for price

$$\frac{p-MC(q)}{p} = -\frac{1}{\varepsilon_{q,p}},$$

$$p = \frac{MC(q)}{1+\frac{1}{\varepsilon_{q,p}}}.$$

which is known as the "inverse elasticity price rule" (IEPR)

- *Example*: If $MC(q) = \$4$ and $\varepsilon_{q,p} = -2$,

$$p = \frac{4}{1+\frac{1}{-2}} = \frac{4}{\frac{1}{2}} = \$8.$$

Multiplant Monopoly

Multiplant Monopoly

- Consider a monopoly producing in two plants (factories),
  - q_1 is the output produced in plant 1,
  - q_2 is the output produced in plant 2,
  - $Q = q_1 + q_2$ represents total output across plants.
- The monopolist maximizes the joint profits from both plants

$$\max_{q_1, q_2} \pi = \pi_1 + \pi_2 = \underbrace{TR_1(q_1, q_2) - TC_1(q_1)}_{\pi_1} + \underbrace{TR_2(q_1, q_2) - TC_2(q_2)}_{\pi_2}$$

$$= [p(q_1, q_2) \times q_1 - TC_1(q_1)] + [p(q_1, q_2) \times q_2 - TC_2(q_2)]$$

$$= p(q_1, q_2) \times (q_1 + q_2) - TC_1(q_1) - TC_2(q_2)$$

Multiplant Monopoly

- Differentiating with respect to q_1, yields

$$\underbrace{p(q_1, q_2) + \frac{\partial p(q_1, q_2)}{\partial q_1}}_{MR_1} = \underbrace{\frac{\partial TC_1(q_1)}{\partial q_1}}_{MC_1},$$

- And differentiating with respect to q_2,

$$\underbrace{p(q_1, q_2) + \frac{\partial p(q_1, q_2)}{\partial q_2}}_{MR_2} = \underbrace{\frac{\partial TC_2(q_2)}{\partial q_2}}_{MC_2},$$

- In the special case that $\frac{\partial p(q_1,q_2)}{\partial q_1} = \frac{\partial p(q_1,q_2)}{\partial q_2}$, $MR_1 = MR_2 = MR$.
- The multiplant monopoly maximizes its joint profits at

$$MR = MC_1 = MC_2.$$

Multiplant Monopoly

- When $\frac{\partial p(q_1,q_2)}{\partial q_1} = \frac{\partial p(q_1,q_2)}{\partial q_2}$,

$$MR_1 = MR_2 = MR.$$

  - This occurs when prices are affected to the same extent when either plant increases its productions, if $p(q_1, q_2) = 300 - q_1 - q_2$.
  - The multiplant monopoly only needs to equate marginal costs across plants.

Multiplant Monopoly

- When $\frac{\partial p(q_1,q_2)}{\partial q_1} \neq \frac{\partial p(q_1,q_2)}{\partial q_2}$,

$$MR_1 \neq MR_2.$$

  - This may occur if $p(q_1, q_2) = 300 - q_1 - 0.5q_2$.
  - The multiplant monopoly maximizes joint profits when $MR_1 = MC_1$ and $MR_2 = MC_2$.

Multiplant Monopoly

- *Example 10.8*: *Multiplant Monopoly*.
  - Consider $p(Q) = 100 - Q = 100 - q_1 - q_2$
  - Assume the e monopolist operates 2 plants
    - Plant 1 (US) with $TC_1(q_1) = 5 + 12q_1 + 6(q_1)^2$
    - Plant 2 (Chile) with $TC_2(q_2) = 5 + 18q_2 + 3(q_2)^2$
  - The monopolist chooses q_1 and q_2 to maximize joint profits from both plants

$$\max_{q_1 \geq 0, q_2 \geq 0} \pi = \pi_1 + \pi_2 = \underbrace{(100 - q_1 - q_2)q_1 - TC_1(q_1)}_{\pi_1}$$
$$+ \underbrace{(100 - q_1 - q_2)q_2 - TC_2(q_2)}_{\pi_2}$$

Multiplant Monopoly

- *Example 10.8* (continued):
  - Differentiating with respect to q_1,

$$100 - 2q_1 - q_2 - 12 - 12q_1 - q_2 = 0,$$

$$88 - 14q_1 - 2q_2,$$

$$q_1 = \frac{44 - q_2}{7}.$$

  - Similarly, differentiating total profits with respect to q_2,

$$100 - q_1 - 2q_2 - 18 - 6q_2 - q_1 = 0,$$

$$82 - 2q_1 - 8q_2,$$

$$q_2 = \frac{41 - q_2}{4}.$$

Multiplant Monopoly

- *Example 10.8* (continued):
  - Inserting the result for q_2 into q_1, we obtain

$$q_1 = \frac{44 - q_2}{7} = \frac{44 - \left(\frac{41 - q_1}{4}\right)}{7},$$

    which simplifies to $7q_1 = \frac{135 - q_1}{4}$, yielding an optimal production in the US plant of $q_1 = 5$ units.
  - The optimal production in the Chilean plant is $q_2 = \frac{41-5}{4} = 9$ units.
  - Aggregate output is $Q = q_1 + q_2 = 5 + 9 = 14$ units.
  - In summary, the monopoly produces a share of $\frac{q_1}{Q} = \frac{5}{9} \cong 0.56$ in the US plant, and $\frac{q_2}{Q} = \frac{4}{9} \cong 0.44$ in the Chilean plant.

Multiplant Monopoly

- The analysis about how the multiplant monopolist determines Q, and how it distributes such production among its plants, q_1 and q_2, is analogous to a "cartel" problem.
- A cartel is a group of firms (equivalent to a monopolist with different plants) coordinating their production decisions to increase their joint profits.
  - *Example*: Organization of the Petroleum-Exporting Countries (OPEC).
    - Some countries have a lower MC (i.e., lower cost of extracting an additional barrel of oil), such as Saudi Arabia.
    - Other countries have higher MC, such as Angola o Venezuela.
    - They coordinate their total production and distribute it among the cartel participants.

Welfare Analysis under Monopoly

Welfare Analysis

- Output is lower under monopoly than under perfectly competitive industries, entailing a higher price.
- Consumer surplus is much smaller than under perfect competition because customers pay more per unit and buy fewer units.
- In contrast, profits are larger.
- However, the firm’s profit gain does not compensate for the loss in consumer surplus, yielding a net loss in social welfare.

Welfare Analysis

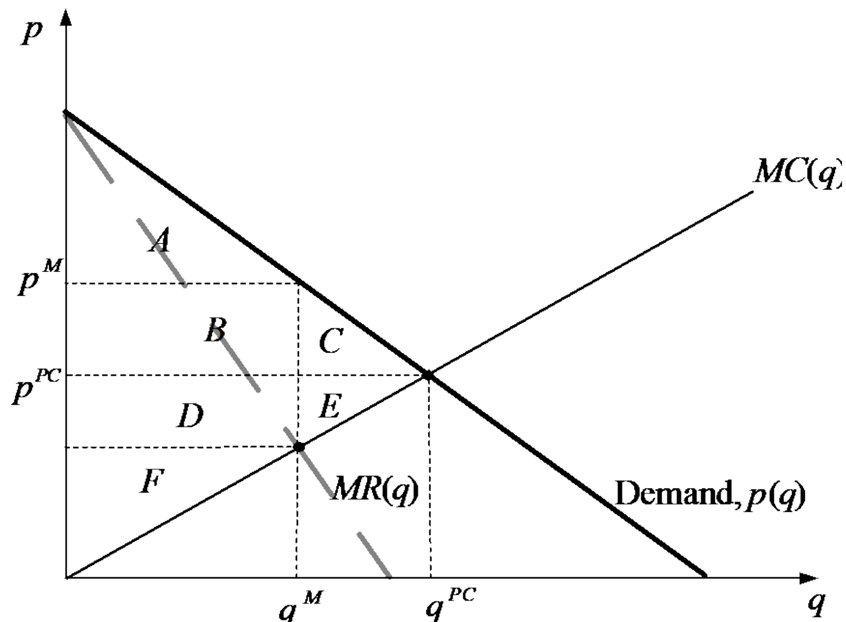


Figure 10.3

Table 10.1

	Perfect Competition	Monopoly	Difference	
Consumer Surplus	$A + B + C$	A	$-B - C$	
Profits	$D + E + F$	$D + F + B$	$B - E$	
Welfare	$A + B + C + D + E + F$	$A + D + F + B$	$-C - E$	} "Deadweight loss"

Welfare Analysis

- *Example 10.9*: *Finding the deadweight loss of a monopoly.*
  - Consider $p(q) = 10 - q$ and $MC(q) = 4$.

Monopoly	Perfect Competition
$q^M = 3$ units	$q^{PC} = 6$ units
$p^M = \$7$	$p^{PC} = \$4$
$CS^M = \frac{1}{2}(10-7)3 = \$4.50$	$CS^{PC} = \frac{1}{2}(10-4)6 = \$18$
$\pi^M = (7 \times 3) - (4 \times 3) = \$9$	$\pi^{PC} = (4 \times 6) - (4 \times 6) = \$0$
$W^M = CS^M + \pi^M$ $= 4.5 + 9 = \$13.50$	$W^{PC} = CS^{PC} + \pi^{PC}$ $= 18 + 0 = \$18$

$$W^{PC} - W^M = 18 - 13.50 = \$4.50.$$

Welfare Analysis

- *Example 10.9* (continued):
  - Deadweight loss under this monopoly is

$$DWL = \frac{1}{2}\underbrace{[p^M - MC(q^M)]}_{\text{height}}\underbrace{(q^{PC} - q^M)}_{\text{base}} = \frac{1}{2}(\$7 - \$4)(6 - 3) = \frac{9}{2} = \$4.50$$

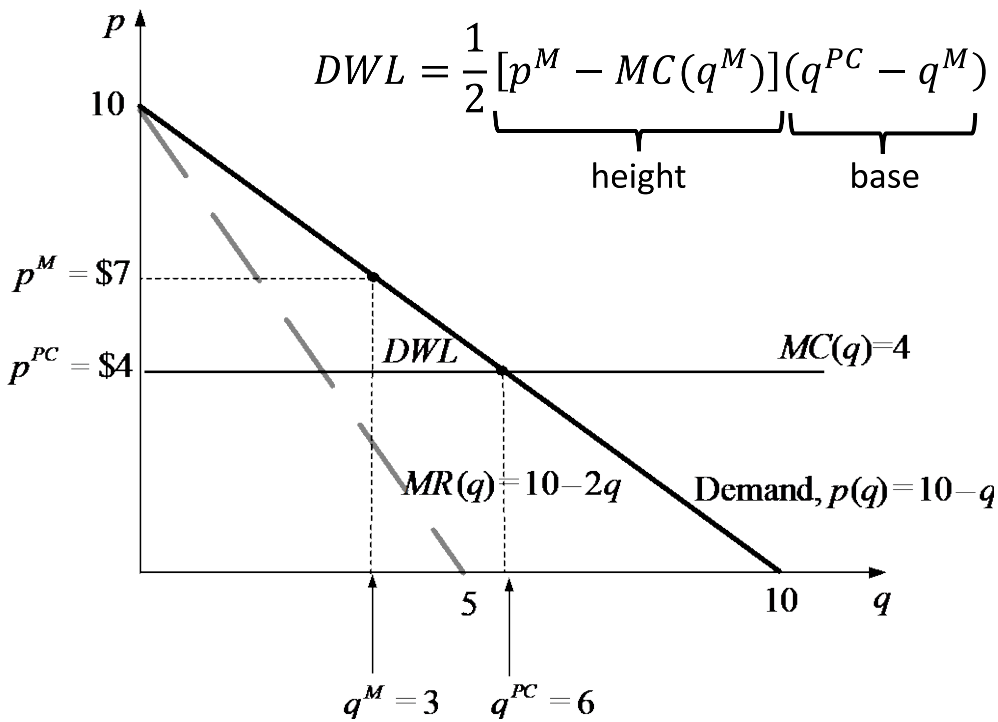


Figure 10.4

Advertising in Monopoly

Advertising in Monopoly

- When investing in advertising, the monopolist faces a trade-off: advertising increases demand but it is costly.
- To find the profit-maximizing amount of advertising, A,

$$\max_{A} \pi = TR - TC - A.$$

- We can rewrite this problem as

$$\begin{aligned}\max_{A} \pi &= (p \times q) - TC(q) - A \\ &= [p \times q(p, A)] - TC[q(p, A)] - A.\end{aligned}$$

- where $q = q(p, A)$ represents the demand function (sales) which is decreasing p, and increasing in A.

Advertising in Monopoly

- Differentiating with respect to the amount of advertising A,

$$p\frac{\partial q(p,A)}{\partial A} - \frac{\partial TC}{\partial q} \cdot \frac{\partial q(p,A)}{\partial A} - 1 = 0.$$

- Rearranging,

$$(p - MC) \cdot \frac{\partial q(p,A)}{\partial A} = 1.$$

Advertising in Monopoly

- Let us define the advertising elasticity of demand, $\varepsilon_{q,A}$, as

$$\varepsilon_{q,A} = \frac{\%\ increase\ in\ q}{\%\ increase\ in\ A} = \frac{\frac{\Delta q}{q}}{\frac{\Delta A}{A}} = \frac{\Delta q}{\Delta A} \cdot \frac{A}{q}.$$

- In the case of a small change in A, the elasticity $\varepsilon_{q,A}$ can be written as $\varepsilon_{q,A} = \frac{\partial q(p,A)}{\partial A} \cdot \frac{A}{q}$.
- Rearranging, we find $\varepsilon_{q,A} \cdot \frac{q}{A} = \frac{\partial q(p,A)}{\partial A}$.

Advertising in Monopoly

- Therefore, we can rewrite the profit-maximizing condition as

$$(p - MC)\underbrace{\varepsilon_{q,A}}_{\frac{\partial q(p,A)}{\partial A}} \cdot \frac{q}{A} = 1.$$

- Dividing both sides by $\varepsilon_{q,A}$ and rearranging,

$$p - MC = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{q}.$$

- Dividing both sides by p, we find

$$\frac{p - MC}{p} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{pq}.$$

Advertising in Monopoly

- From the IERP, we know

$$\frac{p - MC}{p} = -\frac{1}{\varepsilon_{q,p}}.$$

- Hence,

$$-\frac{1}{\varepsilon_{q,p}} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{pq}.$$

$$-\frac{\varepsilon_{q,A}}{\varepsilon_{q,p}} = \frac{A}{pq}.$$

  - The right side represents the advertising-to-sales ratio.
  - For two markets with the same $\varepsilon_{q,p}$, the advertising-to-sales ratio must be larger in the market where demand is more sensitive to advertising (higher $\varepsilon_{q,A}$).

Advertising in Monopoly

- *Example 10.11*: *Monopolist's optimal advertising ratio*.
  - Consider a monopolist with price elasticity of demand of $\varepsilon_{q,p} = -1.5$ and advertising elasticity $\varepsilon_{q,A} = 0.1$.
  - The advertising-to-sales ratio should be

$$\frac{A}{pq} = -\frac{\varepsilon_{q,A}}{\varepsilon_{q,p}}$$
$$= -\frac{0.1}{-1.5} = 0.067.$$

  - Advertising should account for 6.7% of this monopolist's total revenue.

Monopsony

Monopsony

- Monopsony: only one buyer in the market and several sellers.
  - *Examples*: small labor markets, such as a mine or Walmart superstore in a small town.
- The buyer (employer) will be able to pay less for each hour of labor (lower wages) than if it had to compete against other employers, as in a perfectly competitive market.

Monopsony

- Consider a firm (e.g., a coal mine) with production function $q = f(L)$, which:
  - increases with the number of workers hired, $f'(L) > 0$,
  - but at a decreasing rate, $f''(L) < 0$.
- The profits of the coal mine is given by

$$\pi = TR - TC = pq - w(L)L.$$

  - The firm extracts q units of coal, each sold at price p, yielding $TR = pq$.
  - The firm hires L workers, paying each of them a wage of $w(L)$.
    - $w'(L) > 0$, as the firm hires more workers, labor becomes scarce, and a more generous wage must be offered to attract new workers.

Monopsony

- The monopsonist's PMP is

$$\max_{L \geq 0} \; \pi = pq - w(L)L = pf(L) - w(L)L.$$

  - Intuitively, this problem says "choose the number of workers you plan to hire, L, so as to maximize your profits."

- Differentiating with respect to L,

$$pf'(L) - [w(L) + w'(L)L] = 0.$$

- Rearranging,

$$\underbrace{pf'(L)}_{MRP_L} = \underbrace{w(L) + w'(L)L}_{ME_L}.$$

Monopsony

$$\underbrace{pf'(L)}_{MRP_L} = \underbrace{w(L) + w'(L)L}_{ME_L}.$$

- MRP_L ("marginal revenue product" of labor):
  - After hiring 1 more worker (increase in L), the firm produces $f'(L)$ more units of output (e.g., coal), sold at a price p.
- ME_L ("marginal expenditure" on labor). After hiring 1 more worker, the firm experiences an increase in cost:
  - This extra worker must be paid $w(L)$.
  - The additional worker is only attracted to the job if the firm offers her a higher salary because labor becomes scarcer. Such a wage increase, $w'(L)$, must be passed on to all existing worker, entailing a cost increase of $w'(L)L$.

Monopsony

- *Example 10.12*: *Finding optimal L in monopsony*.
    - Consider a coal company in a small town with production function $q = 100 \times \ln(L)$.
    - It faces an international perfectly competitive price of coal, $p = \$8$.
    - Assume the supply curve for labor is $w(L) = 3 + \frac{1}{2}L$. Then,

$$MRP_L = pf'(L) = 8 \times 100\frac{1}{L} = \frac{800}{L}.$$

$$ME_L = w(L) + w'(L)L = \left(3 + \frac{1}{2}L\right) + \frac{1}{2}L = 3 + L.$$

Monopsony

- *Example 10.12* (continued):

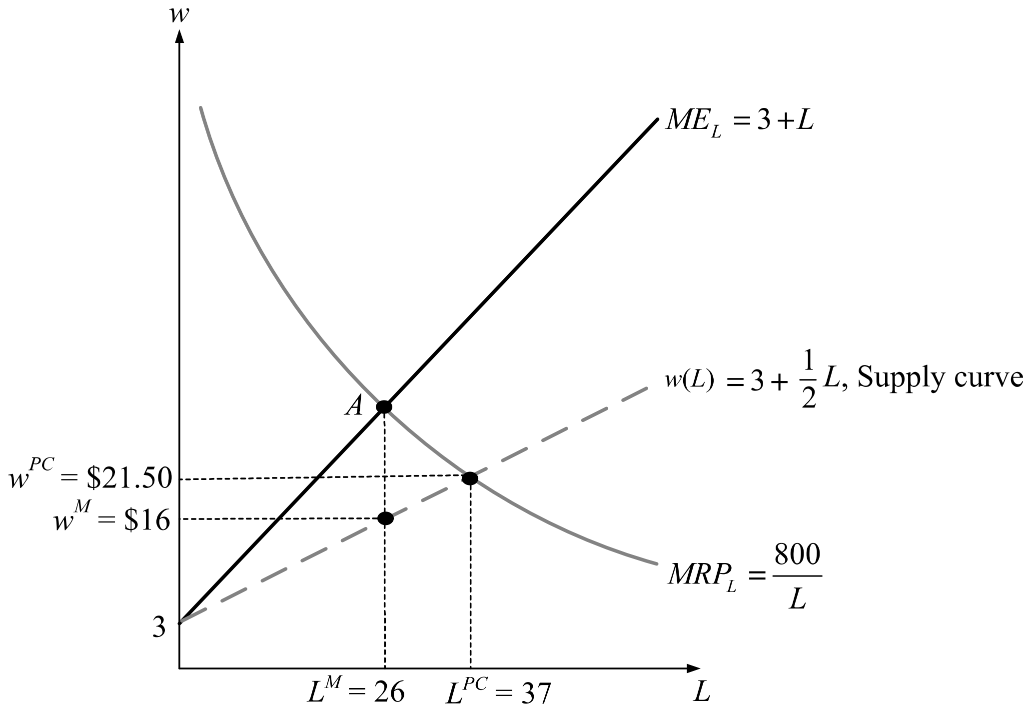


Figure 10.5

Monopsony

- *Example 10.12* (continued):
  - Setting $MRP_L = ME_L$,

$$\frac{800}{L} = 3 + L,$$

    which expanding yields $800 = 3L + L^2$ or

$$L^2 + 3L - 800 = 0.$$

  - Solving for L, we find $L = -29.82$ and $L = 26.82$. Because the firm must hire a positive number of workers (or zero), we find that $L^M = 26$ workers is optimal.
  - At $L^M = 26$, wages become $w(26) = 3 + 26 \times \frac{1}{2} = \$16$.

Monopsony

- *Example 10.12* (continued):
  - Under a perfectly competitive labor market, we have $MRP_L = w(L)$, that is,

$$\frac{800}{L} = 3 + \frac{1}{2}L,$$

    which expanding yields $800 = 3L + \frac{L^2}{2}$.
  - Solving for L, we obtain $L = -43.11$ and $L = 37.11$. Then $L^{PC} = 37$ is the optimal number of workers.
  - At $L^{PC} = 37$, wages become $w(37) = 3 + \frac{1}{2}37 = \$21.5$.