

Agent i

In[6]:= **FullSimplify** $\left[D\left[a \operatorname{Log}\left[w_i - g_i\right] + b \operatorname{Log}\left[g_i + g_j\right] + c \operatorname{Log}\left[g_i\right], g_i\right]\right]$

Out[6]=
$$\frac{c}{g_i} + \frac{b}{g_i + g_j} + \frac{a}{g_i - w_i}$$

In[26]:= **FullSimplify** $\left[\frac{c}{g_i} + \frac{b}{g_i + g_j} + \frac{a}{g_i - w_i} /. \left\{a \rightarrow 1, b \rightarrow \frac{1}{3}, c \rightarrow \frac{1}{4}, w_i \rightarrow 10\right\}\right]$

Out[26]=
$$\frac{1}{-10 + g_i} + \frac{1}{4 g_i} + \frac{1}{3 (g_i + g_j)}$$

In[27]:= **FullSimplify** $\left[\operatorname{Solve}\left[\frac{1}{-10 + g_i} + \frac{1}{4 g_i} + \frac{1}{3 (g_i + g_j)} == 0, g_i\right]\right]$

Out[27]=
$$\left\{\left\{g_i \rightarrow \frac{1}{38} \left(70 - 15 g_j - \sqrt{5} \sqrt{980 + 9 g_j (4 + 5 g_j)}\right)\right\}, \right. \\ \left.\left\{g_i \rightarrow \frac{1}{38} \left(70 - 15 g_j + \sqrt{5} \sqrt{980 + 9 g_j (4 + 5 g_j)}\right)\right\}\right\}$$

Agent j

In[11]:= **FullSimplify** $\left[D\left[a \operatorname{Log}\left[w_j - g_j\right] + b \operatorname{Log}\left[g_i + g_j\right] + c \operatorname{Log}\left[g_j\right], g_j\right]\right]$

Out[11]=
$$\frac{c}{g_j} + \frac{b}{g_i + g_j} + \frac{a}{g_j - w_j}$$

In[12]:= **FullSimplify** $\left[\frac{c}{g_j} + \frac{b}{g_i + g_j} + \frac{a}{g_j - w_j} /. \left\{a \rightarrow 1, b \rightarrow \frac{1}{3}, c \rightarrow \frac{1}{5}, w_j \rightarrow 10\right\}\right]$

Out[12]=
$$\frac{1}{-10 + g_j} + \frac{1}{5 g_j} + \frac{1}{3 (g_i + g_j)}$$

In[13]:= **FullSimplify** $\left[\operatorname{Solve}\left[\frac{1}{-10 + g_j} + \frac{1}{5 g_j} + \frac{1}{3 (g_i + g_j)} == 0, g_j\right]\right]$

Out[13]=
$$\left\{\left\{g_j \rightarrow \frac{1}{23} \left(40 - 9 g_i - \sqrt{1600 - 30 g_i + 81 g_i^2}\right)\right\}, \left\{g_j \rightarrow \frac{1}{23} \left(40 - 9 g_i + \sqrt{1600 - 30 g_i + 81 g_i^2}\right)\right\}\right\}$$

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In[28]:= FullSimplify[
  
$$\frac{1}{38} \left( 70 - 15 g_j + \sqrt{5} \sqrt{980 + 9 g_j (4 + 5 g_j)} \right) /. g_j \rightarrow \frac{1}{23} \left( 40 - 9 g_i + \sqrt{1600 - 30 g_i + 81 g_i^2} \right) ]$$

Out[28]:=

$$\frac{1}{38} \left( 70 - \frac{15}{23} \left( 40 - 9 g_i + \sqrt{1600 - 30 g_i + 81 g_i^2} \right) + \right.$$


$$\left. \sqrt{5} \sqrt{\left( 980 + \frac{9}{529} \left( 40 - 9 g_i + \sqrt{1600 - 30 g_i + 81 g_i^2} \right) \left( 292 - 45 g_i + 5 \sqrt{1600 - 30 g_i + 81 g_i^2} \right) \right)} \right)$$

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In[29]:= FullSimplify[Solve[
$$\frac{1}{38} \left( 70 - \frac{15}{23} \left( 40 - 9 g_i + \sqrt{1600 - 30 g_i + 81 g_i^2} \right) + \right.$$


$$\left. \sqrt{5} \sqrt{\left( 980 + \frac{9}{529} \left( 40 - 9 g_i + \sqrt{1600 - 30 g_i + 81 g_i^2} \right) \right.} \right.$$


$$\left. \left. \left( 292 - 45 g_i + 5 \sqrt{1600 - 30 g_i + 81 g_i^2} \right) \right) \right] = g_i, g_i ] // N$$

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Out[29]=
{ { g_i -> 2.9958 } }
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In[30]:= FullSimplify[
$$\frac{1}{23} \left( 40 - 9 g_i + \sqrt{1600 - 30 g_i + 81 g_i^2} \right) /. g_i \rightarrow 2.9958049427497264` ]$$

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Out[30]=
2.62329
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Aggregate donation

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In[31]:= FullSimplify[2.9958049427497264` + 2.6232880533953336` ]
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Out[31]=
5.61909
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