Advanced Microeconomic Theory

Chapter 6: Partial and General Equilibrium

Outline

- Partial Equilibrium Analysis
- General Equilibrium Analysis
- Comparative Statics
- Welfare Analysis

Partial Equilibrium Analysis

- In a competitive equilibrium (CE), all agents must select an optimal allocation given their resources:
  - Firms choose profit-maximizing production plans given their technology;
  - Consumers choose utility-maximizing bundles given their budget constraint.
- A competitive equilibrium allocation will emerge at a price that makes consumers' purchasing plans to coincide with the firms' production decision.

Partial Equilibrium Analysis

- ***Firm***:
  - Given the price vector p^*, firm j's equilibrium output level q_j^* must solve

$$\max_{q_j \geq 0} \quad p^* q_j - c_j(q_j)$$

    which yields the necessary and sufficient condition

$$p^* \leq c_j'(q_j^*), \text{ with equality if } q_j^* > 0$$

  - That is, every firm j produces until the point in which its marginal cost, $c_j'(q_j^*)$, coincides with the current market price.

Partial Equilibrium Analysis

- ***Consumers***:
  - Consider a quasilinear utility function

$$u_i(m_i, x_i) = m_i + \phi_i(x_i)$$

    where m_i denotes the numeraire, and $\phi_i'(x_i) > 0$, $\phi_i''(x_i) < 0$ for all $x_i > 0$.
  - Normalizing, $\phi_i(0) = 0$. Recall that with quasilinear utility functions, the wealth effects for all non-numeraire commodities are zero.

Partial Equilibrium Analysis

– Consumer i's UMP is

$$\max_{m_i \in \mathbb{R}_+,\, x_i \in \mathbb{R}_+} \quad m_i + \phi_i(x_i)$$

$$\text{s.t.} \quad \underbrace{m_i + p^* x_i}_{\text{Total expend.}} \leq \underbrace{w_{m_i} + \sum_{j=1}^{J} \theta_{ij} (\underbrace{p^* q_j^* - c_j(q_j^*)}_{\text{Profits}})}_{\text{Total resources (endowment+profits)}}$$

– The budget constraint must hold with equality (by Walras' law). Hence,

$$m_i = -p^* x_i + \left[w_{m_i} + \sum_{j=1}^{J} \theta_{ij} \left(p^* q_j^* - c_j(q_j^*) \right) \right]$$

Partial Equilibrium Analysis

– Substituting the budget constraint into the objective function,

$$\max_{x_i \in \mathbb{R}_+} \quad \phi_i(x_i) - p^* x_i + \left[w_{m_i} + \sum_{j=1}^{J} \theta_{ij} \left(p^* q_j^* - c_j(q_j^*) \right) \right]$$

– FOCs wrt x_i yields

$\phi_i'(x_i^*) \leq p^*$, with equality if $x_i^* > 0$

– That is, consumer increases the amount he buys of good x until the point in which the marginal utility he obtains exactly coincides with the market price he has to pay for it.

Partial Equilibrium Analysis

– Hence, an allocation $(x_1^*, x_2^*, \dots, x_I^*, q_1^*, q_2^*, \dots, q_J^*)$ and a price vector $p^* \in \mathbb{R}^L$ constitute a CE if:

$$p^* \leq c_j'(q_j^*), \text{ with equality if } q_j^* > 0$$

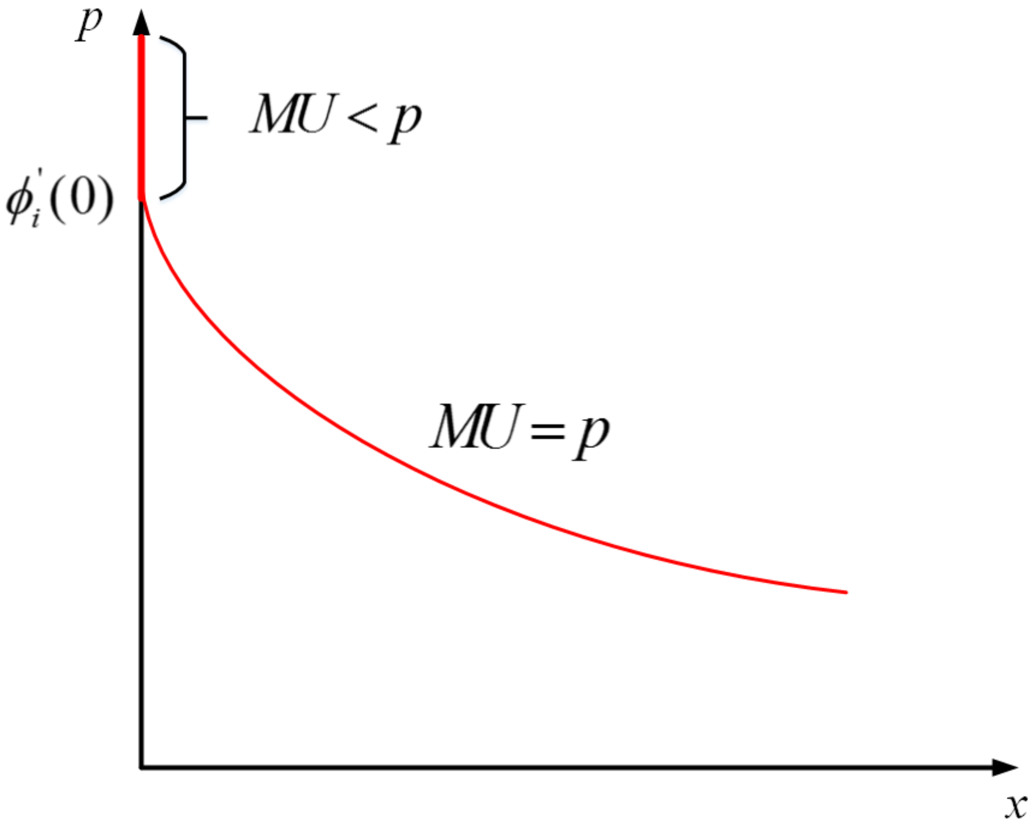
$$\phi_i'(x_i^*) \leq p^*, \text{ with equality if } x_i^* > 0$$

$$\sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*$$

– Note that the these conditions do not depend upon the consumer's initial endowment.

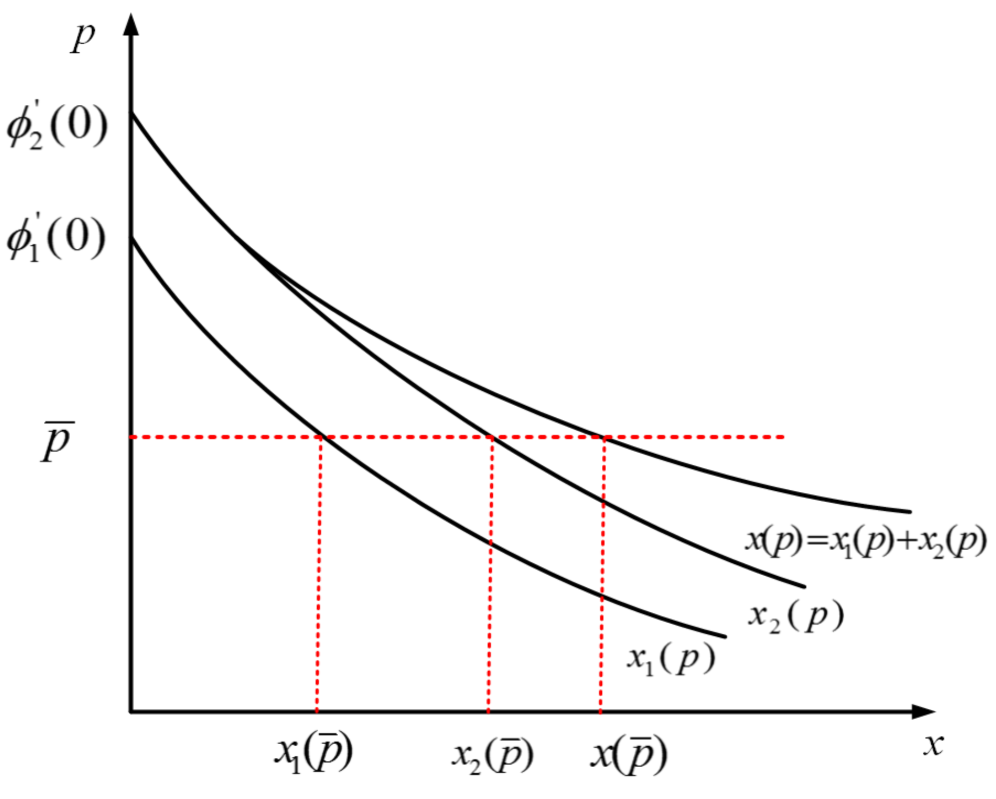
Partial Equilibrium Analysis

- The individual demand curve, where $\phi_i'(x_i^*) \leq p^*$



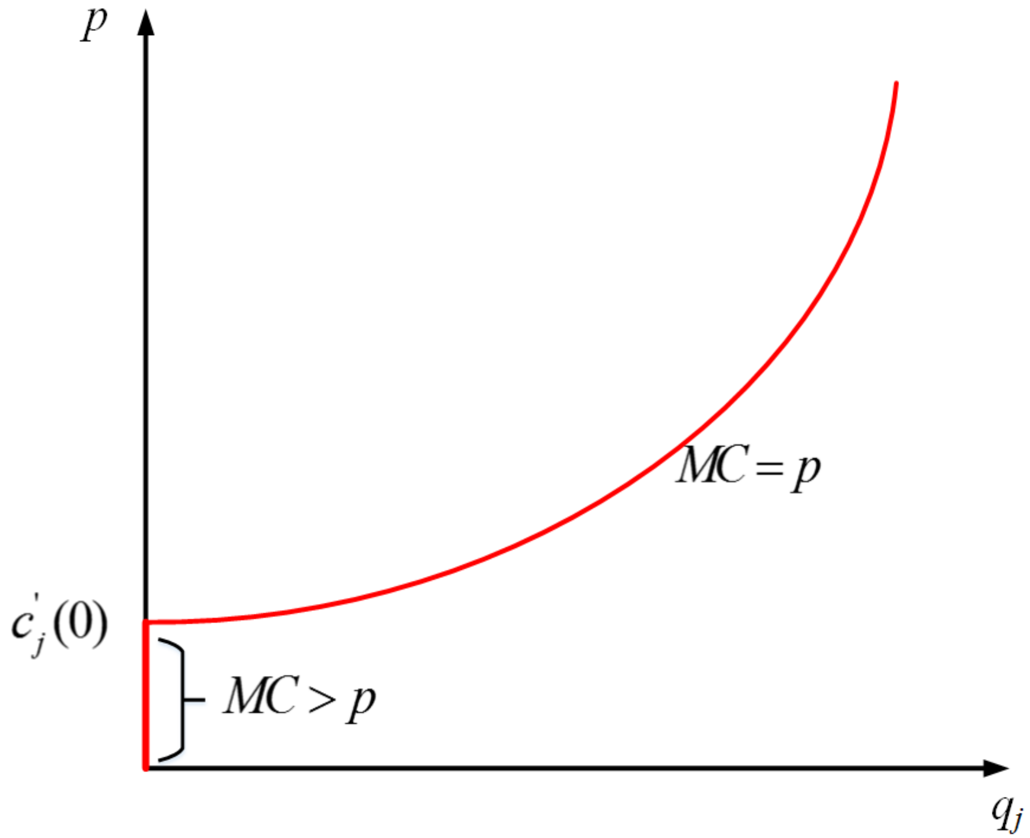
Partial Equilibrium Analysis

- Horizontally summing individual demand curves yields the aggregate demand curve.



Partial Equilibrium Analysis

- The individual supply curve, where $p^* \leq c_j'(q_j^*)$



Partial Equilibrium Analysis

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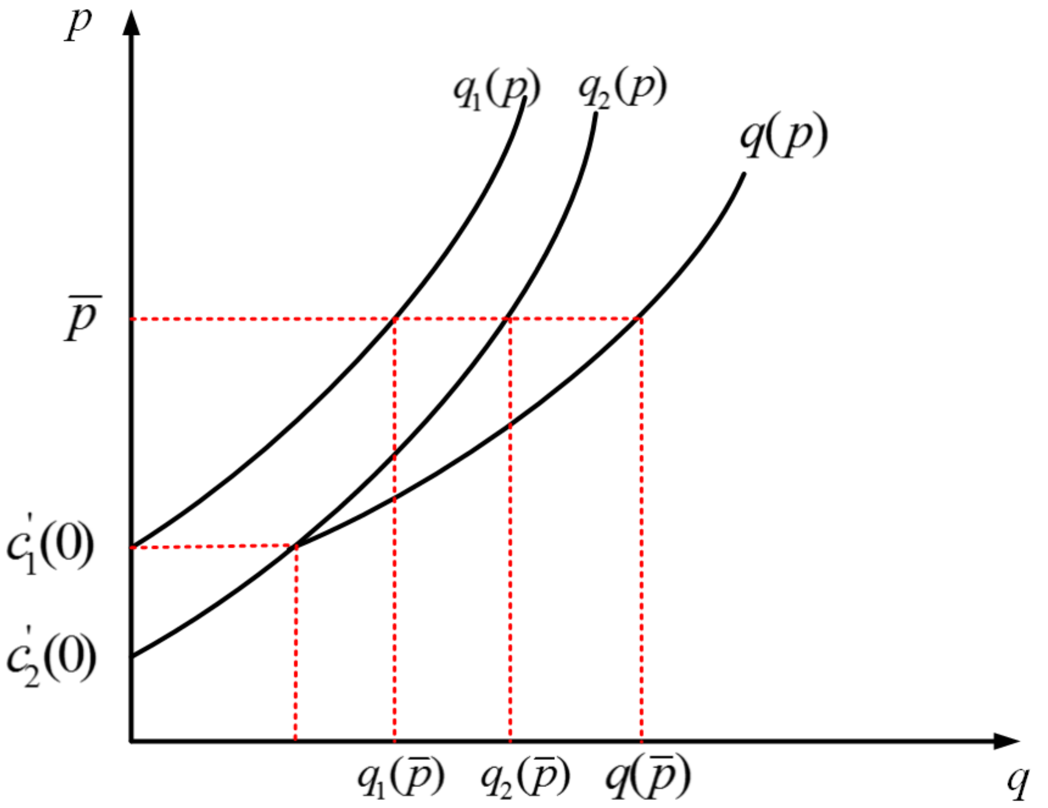
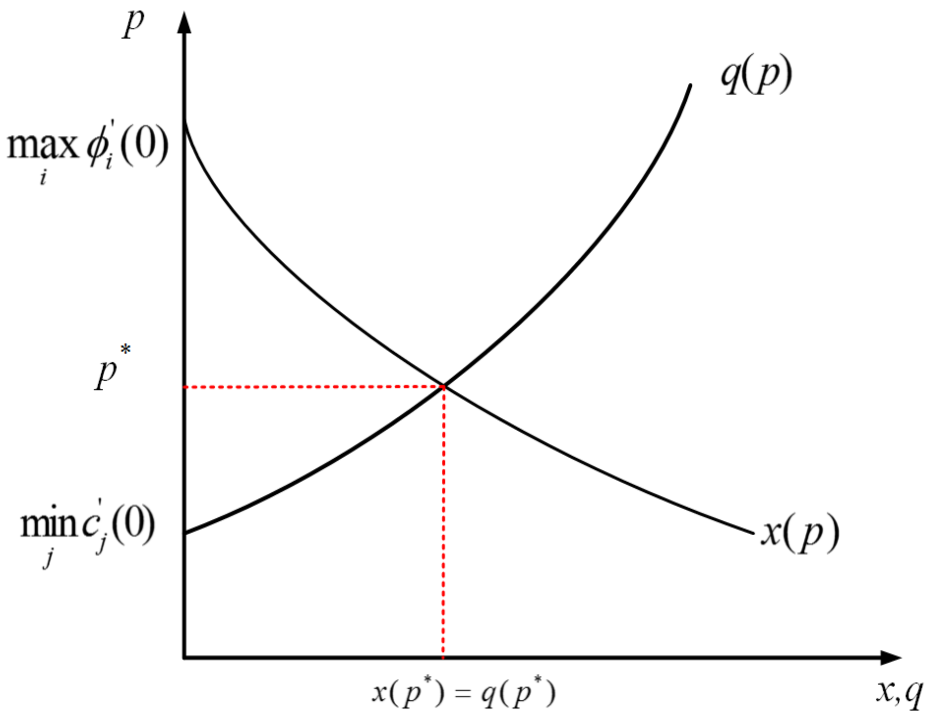


Partial Equilibrium Analysis

- Superimposing aggregate demand and aggregate supply curves, we obtain the CE allocation of good x.
- To guarantee that a CE exists, the equilibrium price p^* must satisfy

$$\max_i \phi_i'(0) \geq p^* \geq \min_j c_j'(0)$$



Partial Equilibrium Analysis

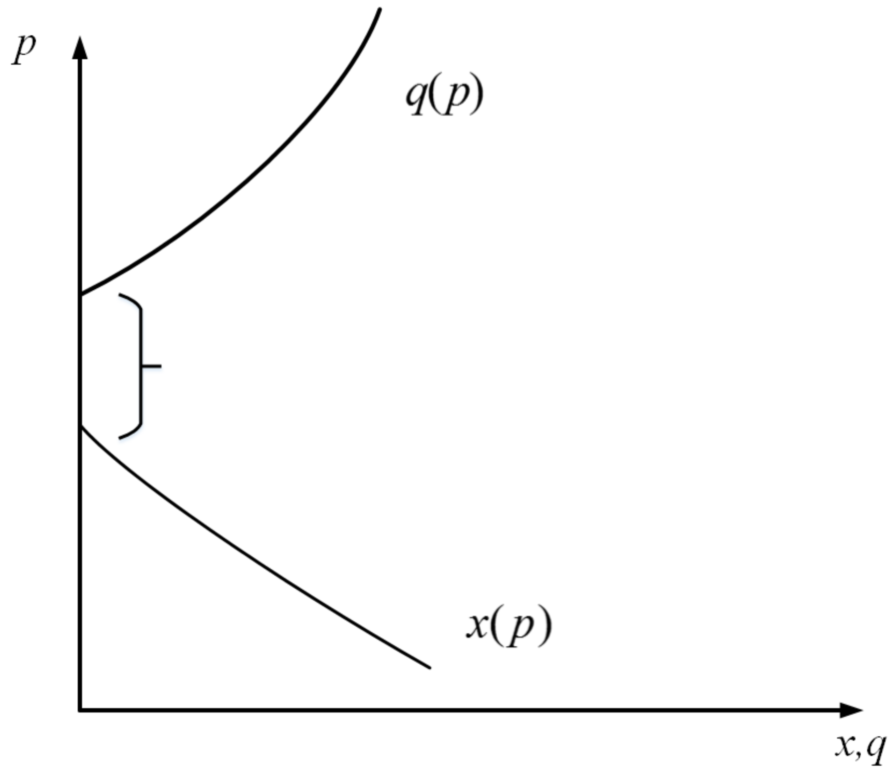
- Also, since $\phi_i'(x_i)$ is downward sloping in x_i, and $c_j'(q_i)$ is upward sloping in q_i, then aggregate demand and supply cross at a unique point.
  – Hence, the CE allocation is unique.

Partial Equilibrium Analysis

- If we have

  $$\max_i \phi_i'(0) < \min_j c_j'(0),$$

  then there is *no* positive production or consumption of good x.



Partial Equilibrium Analysis

- ***Example 6.1***:
  - Assume a perfectly competitive industry consisting of two types of firms: 100 firms of type *A* and 30 firms of type *B.*
  - Short-run supply curve of type *A* firm is
    $$s_A(p) = 2p$$
  - Short-run supply curve of type *B* firm is
    $$s_B(p) = 10p$$
  - The Walrasian market demand curve is
    $$x(p) = 5000 - 500p$$

Partial Equilibrium Analysis

- ***Example 6.1*** (continued):
  – Summing the individual supply curves of the 100 type-*A* firms and the 30 type-*B* firms,
  $$S(p) = 100 \cdot 2p + 30 \cdot 10p = 500p$$
  – The short-run equilibrium occurs at the price at which quantity demanded equals quantity supplied,
  $$5000 - 500p = 500p, \text{ or } p = 5$$
  – Each type-*A* firm supplies: $s_A(p) = 2 \cdot 5 = 10$
  – Each type-*B* firm supplies: $s_B(p) = 10 \cdot 5 = 50$

Comparative Statics

Comparative Statics

- Let us assume that the consumer's preferences are affected by a vector of parameters $\alpha \in \mathbb{R}^M$, where $M \leq L$.
  - Then, consumer i's utility from good x is $\phi_i(x_i, \alpha)$.
- Similarly, firms' technology is affected by a vector of parameters $\beta \in \mathbb{R}^S$, where $S \leq L$.
  - Then, firm j's cost function is $c_j(q_j, \beta)$.

- Notation:
  - $\hat{p}_i(p, t)$ is the effective price paid by the consumer
  - $\hat{p}_j(p, t)$ is the effective price received by the firm
  - Per unit tax: $\hat{p}_i(p, t) = p + t$.
    - Example: $t = \$2$, regardless of the price p
  - Ad valorem tax (sales tax): $\hat{p}_i(p, t) = p + pt = p(1 + t)$
    - Example: $t = 0.1$ (10%).

Comparative Statics

- If consumption and production are strictly positive in the CE, then

$$\phi_i'(x_i^*, \alpha) = \hat{p}_i(p^*, t) \text{ for every consumer } i$$

$$c_j'(q_j^*, \beta) = \hat{p}_j(p^*, t) \text{ for every firm } j$$

$$\sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*$$

- Then we have $I + J + 1$ equations, which depend on parameter values α, β and t.
- In order to understand how x_i^* or q_j^* depends on parameters α and β, we can use the ***Implicit Function Theorem***.
  - The above functions have to be differentiable.

Comparative Statics

- ***Implicit Function Theorem***:
  - Let $u(x, y)$ be a utility function, where x and y are amounts of two goods.
  - If $\frac{\partial u(\bar{x},\bar{y})}{\partial x} \neq 0$ when evaluated at $(\bar{x}, \bar{y})$, then

$$\frac{\partial u(\bar{x},\bar{y})}{\partial x} dx + \frac{\partial u(\bar{x},\bar{y})}{\partial y} dy = 0$$

which yields

$$\frac{dy(\bar{x})}{dx} = -\frac{\frac{\partial u(\bar{x},\bar{y})}{\partial x}}{\frac{\partial u(\bar{x},\bar{y})}{\partial y}}$$

Comparative Statics

– Similarly, if $\frac{\partial u(\bar{x},\bar{y})}{\partial y} \neq 0$ when evaluated at $(\bar{x}, \bar{y})$, then

$$\frac{dx(\bar{y})}{dy} = -\frac{\frac{\partial u(\bar{x},\bar{y})}{\partial y}}{\frac{\partial u(\bar{x},\bar{y})}{\partial x}}$$

for all $(\bar{x}, \bar{y})$.

Comparative Statics

- Similarly, if $u(x, \alpha)$ describes the consumption of a single good x,where α determines the consumer's preference for x, and $\frac{\partial u(x,\alpha)}{\partial \alpha} \neq 0$, then

$$\frac{dx(\alpha)}{d\alpha} = -\frac{\frac{\partial u(x,\alpha)}{\partial \alpha}}{\frac{\partial u(x,\alpha)}{\partial x}}$$

- The left-hand side is unknown
- The right-hand side is, however, easier to find.

Comparative Statics

- ***Sales tax*** (Example 6.2):
  - The expression of the aggregate demand is now $x(p+t)$, because the effective price that the consumer pays is actually $p+t$.
  - In equilibrium, the market price after imposing the tax, $p^*(t)$, must hence satisfy
$$x(p^*(t)+t) = q(p^*(t))$$
  - if the sales tax is marginally increased, and functions are differentiable at $p = p^*(t)$,
$$x'(p^*(t)+t)\cdot(p^{*\prime}(t)+1) = q'(p^*(t))\cdot p^{*\prime}(t)$$

Comparative Statics

- Rearranging, we obtain

$$p^{*\prime}(t) \cdot \left[x'(p^*(t)+t) - q'\left(p^*(t)\right)\right] = -x'(p^*(t)+t)$$

- Hence,

$$p^{*\prime}(t) = -\frac{x'(p^*(t)+t)}{x'(p^*(t)+t) - q'\left(p^*(t)\right)}$$

- Since $x(p)$ is decreasing in prices, $x'(p^*(t)+t) < 0$, and $q(p)$ is increasing in prices, $q'\left(p^*(t)\right) > 0$,

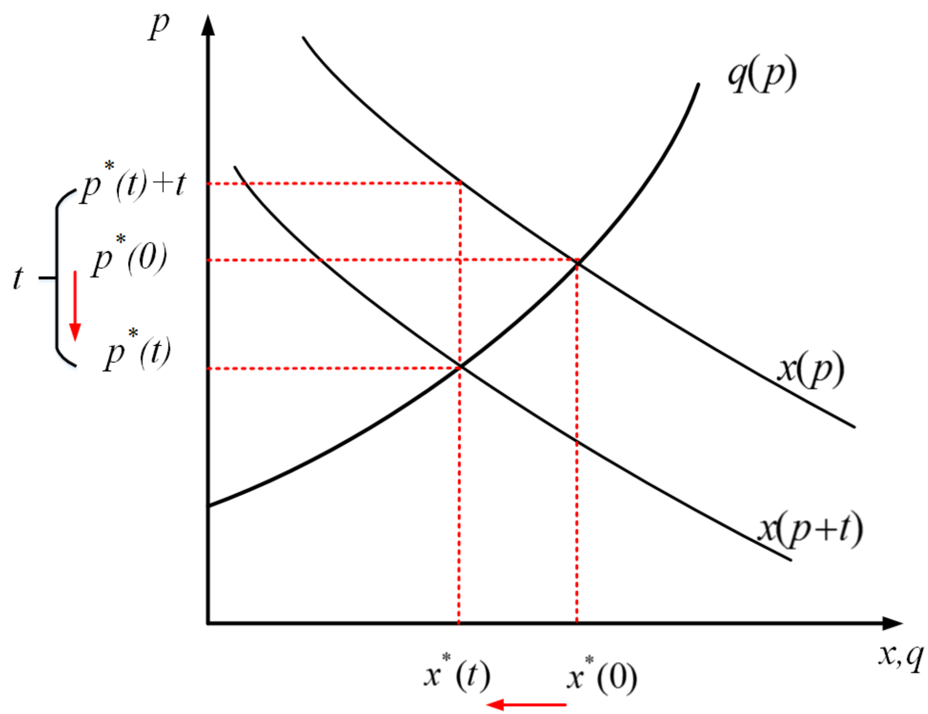
$$p^{*\prime}(t) = -\frac{x'(p^*(t)+t)}{\underbrace{x'(p^*(t)+t)}_{-} - \underbrace{q'\left(p^*(t)\right)}_{+}} = -\frac{-}{-} = -$$

Comparative Statics

– Hence, $p^{*\prime}(t) < 0$.
– Moreover, $p^{*\prime}(t) \in (-1,0]$.
– Therefore, $p^{*}(t)$ decreases in t.
  - That is, the price received by producers falls in the tax, but less than proportionally.
– Additionally, since $p^{*}(t) + t$ is the price paid by consumers, then $p^{*\prime}(t) + 1$ is the marginal increase in the price paid by consumers when the tax marginally increases.
  - Since $p^{*\prime}(t) \geq 1$, then $p^{*\prime}(t) + 1 \geq 0$, and consumers' cost of the product also raises less than proportionally.

Comparative Statics

- *No tax:*
  - CE occurs at $p^*(0)$ and $x^*(0)$
- *Tax:*
  - x^* decreases from $x^*(0)$ to $x^*(t)$
  - Consumers now pay $p^*(t) + t$
  - Producers now receive $p^*(t)$ for the $x^*(t)$ units they sell.



Comparative Statics

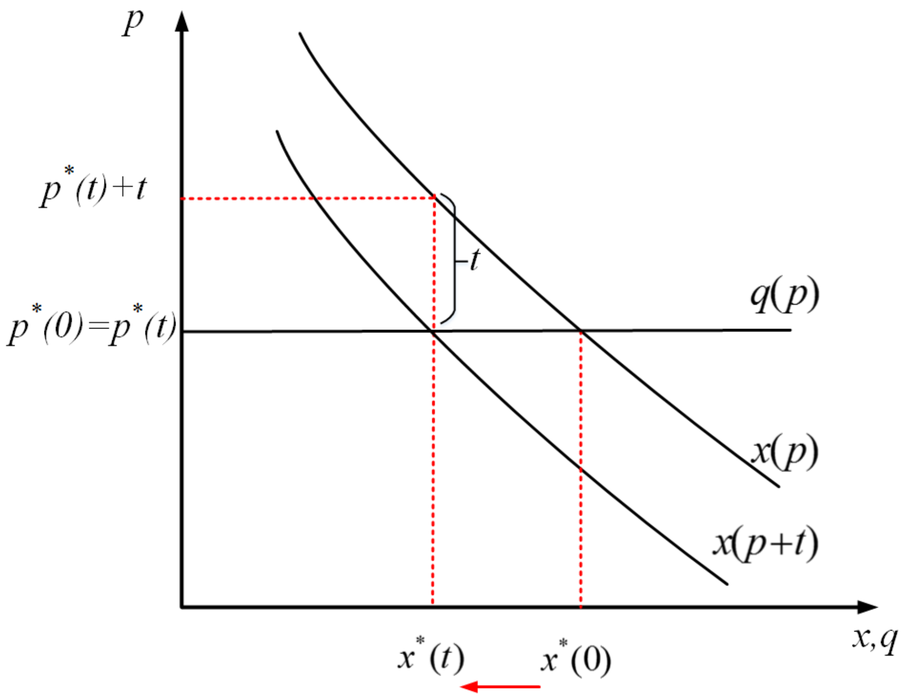
- ***Sales Tax*** (Extreme Cases):
  a) *The supply is very responsive to price changes, i.e., $q'\left(p^*(t)\right)$ is large.*

$$p^{*\prime}(t) = -\frac{x'(p^*(t)+t)}{x'(p^*(t)+t)-q'\left(p^*(t)\right)} \to 0$$

  – Therefore, $p^{*\prime}(t) \to 0$, and the price received by producers does not fall.
  – However, consumers still have to pay $p^*(t) + t$.
    - A marginal increase in taxes therefore provides an increase in the consumer's price of
    $$p^{*\prime}(t) + 1 = 0 + 1 = 1$$
    - The tax is solely borne by consumers.

Comparative Statics

- A very elastic supply curve
  - The price received by producers almost does not fall.
  - But, the price paid by consumers increases by exactly the amount of the tax.



Comparative Statics

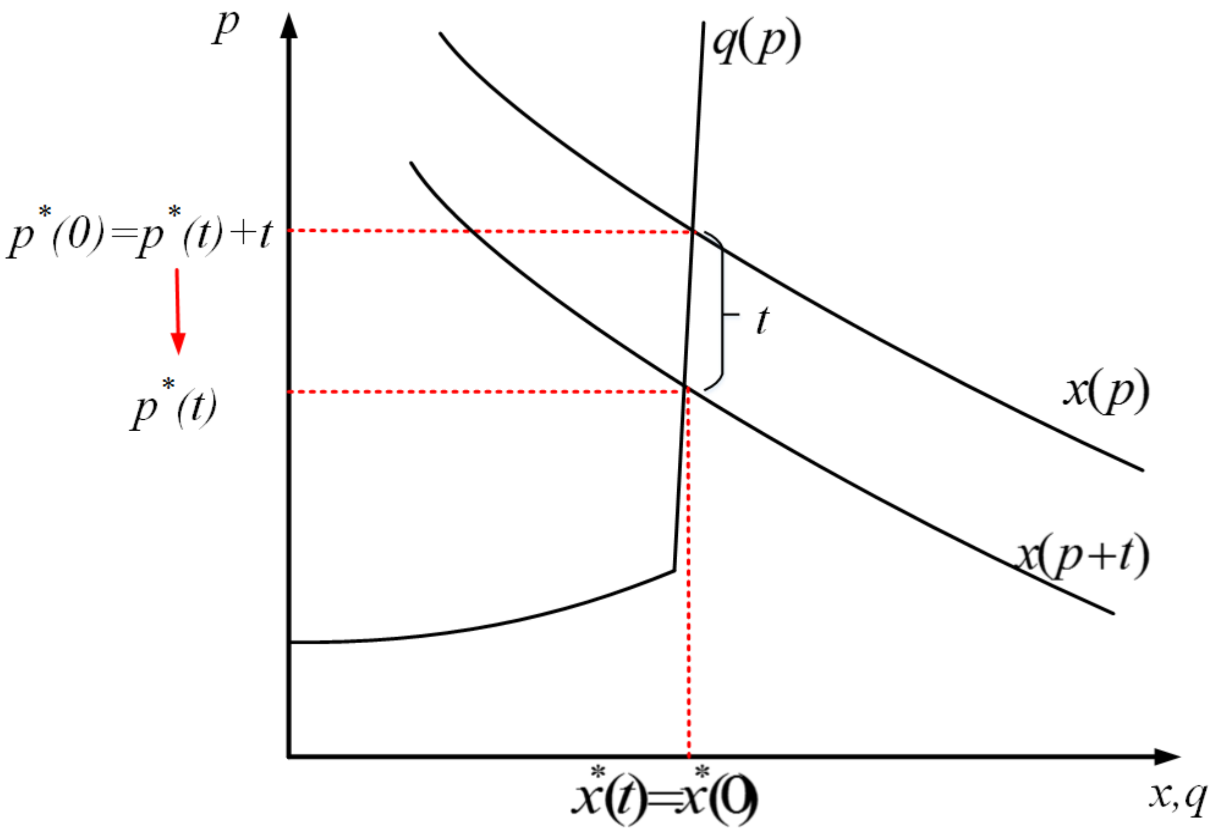
b) *The supply is not responsive to price changes, i.e.,* $q'\left(p^*(t)\right)$ *is close to zero.*

$$p^{*\prime}(t) = -\frac{x'(p^*(t)+t)}{x'(p^*(t)+t)-q'\left(p^*(t)\right)} = -1$$

- Therefore, $p^{*\prime}(t) = -1$, and the price received by producers falls by $1 for every extra dollar in taxes.
  - Producers bear most of the tax burden
- In contrast, consumers pay $p^*(t) + t$
  - A marginal increase in taxes produces an increase in the consumer's price of
  $$p^{*\prime}(t) + 1 = -1 + 1 = 0$$
  - Consumers do not bear tax burden at all.

Comparative Statics

- Inelastic supply curve



Comparative Statics

- ***Example 6.3***:
  - Consider a competitive market in which the government will be imposing an ad valorem tax of t.
  - Aggregate demand curve is $x(p) = Ap^{\varepsilon}$, where $A > 0$ and $\varepsilon < 0$, and aggregate supply curve is $q(p) = ap^{\gamma}$, where $a > 0$ and $\gamma > 0$.
  - Let us evaluate how the equilibrium price is affected by a marginal increase in the tax.

Comparative Statics

- ***Example 6.3*** (continued):
  - The change in the price received by producers at $t = 0$ is

$$p^{*\prime}(0) = -\frac{x'(p^*)}{x'(p^*) - q'(p^*)}$$

$$= -\frac{A\varepsilon p^{*\varepsilon-1}}{A\varepsilon p^{*\varepsilon-1} - a\gamma p^{*\gamma-1}} = -\frac{A\varepsilon p^{*\varepsilon}}{A\varepsilon p^{*\varepsilon} - a\gamma p^{*\gamma}}$$

$$= -\frac{\varepsilon x(p^*)}{\varepsilon x(p^*) - \gamma q(p^*)} = -\frac{\varepsilon}{\varepsilon - \gamma}$$

  - The change in the price paid by consumers at $t = 0$ is

$$p^{*\prime}(0) + 1 = -\frac{\varepsilon}{\varepsilon - \gamma} + 1 = -\frac{\gamma}{\varepsilon - \gamma}$$

Comparative Statics

- ***Example 6.3*** (continued):
  - When $\gamma = 0$ (i.e., supply is perfectly inelastic), the price paid by consumers in unchanged, and the price received by producers decreases be the amount of the tax.
    - That is, producers bear the full effect of the tax.
  - When $\varepsilon = 0$ (i.e., demand is perfectly inelastic), the price received by producers is unchanged and the price paid by consumers increases by the amount of the tax.
    - That is, consumers bear the full burden of the tax.

Comparative Statics

- ***Example 6.3*** (continued):
  - When $\varepsilon \rightarrow -\infty$ (i.e., demand is perfectly elastic), the price paid by consumers is unchanged, and the price received by producers decreases by the amount of the tax.
  - When $\gamma \rightarrow +\infty$(i.e., supply is perfectly elastic), the price received by producers is unchanged and the price paid by consumers increases by the amount of the tax.

Welfare Analysis

Welfare Analysis

- Let us now measure the changes in the aggregate social welfare due to a change in the competitive equilibrium allocation.
- Consider the aggregate surplus

$$S = \sum_{i=1}^{I} \phi_i(x_i) - \sum_{j=1}^{J} c_j(q_j)$$

- Take a differential change in the quantity of good k that individuals consume and that firms produce such that $\sum_{i=1}^{I} dx_i = \sum_{j=1}^{J} dq_j$.
- The change in the aggregate surplus is

$$dS = \sum_{i=1}^{I} \phi_i'(x_i)dx_i - \sum_{j=1}^{J} c_j'(q_j)dq_j$$

Welfare Analysis

- Since
  - $\phi_i'(x_i) = P(x)$ for all consumers; and
    - That is, every individual consumes until MB=p.
  - $c_j'(q_j) = C'(q)$ for all firms
    - That is, every firm's MC coincides with aggregate MC)

then the change in surplus can be rewritten as

$$dS = \sum_{i=1}^{I} P(x)dx_i - \sum_{j=1}^{J} C'(q)dq_j$$

$$= P(x)\sum_{i=1}^{I} dx_i - C'(q)\sum_{j=1}^{J} dq_j$$

Welfare Analysis

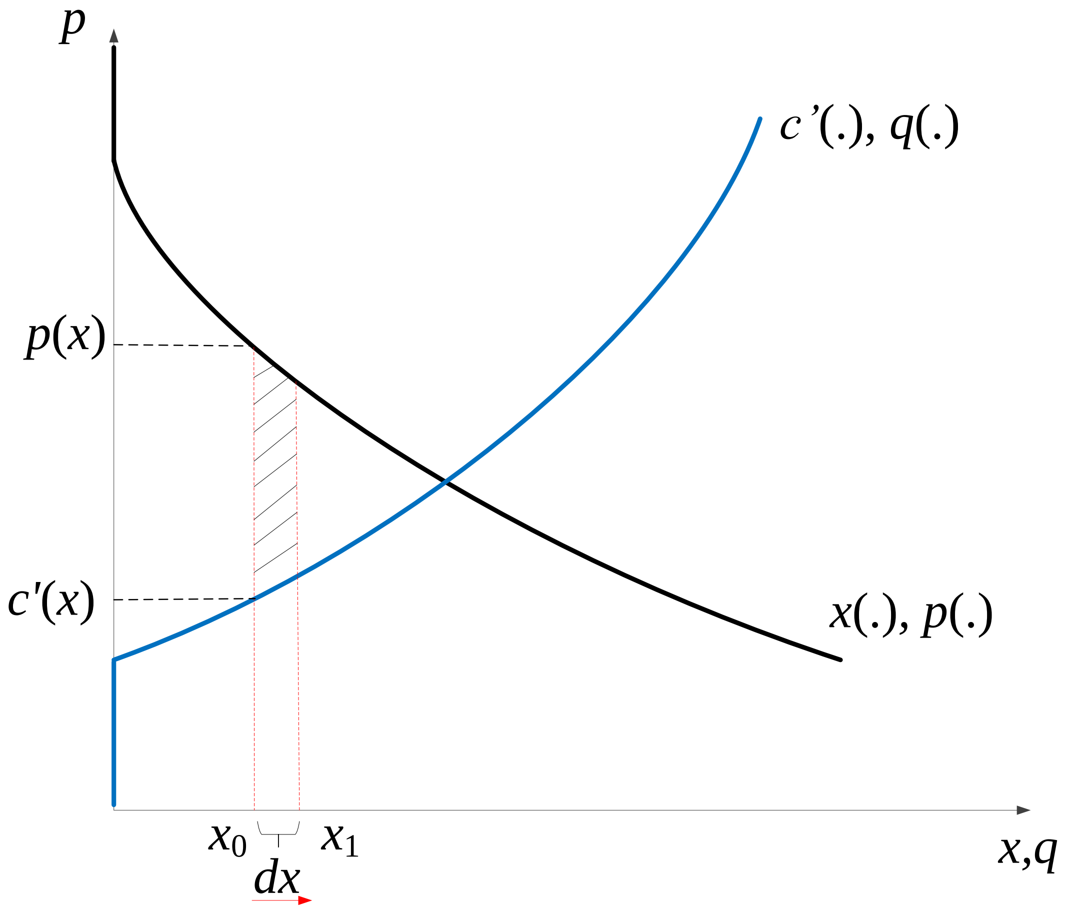
- But since $\sum_{i=1}^{I} dx_i = \sum_{j=1}^{J} dq_j = dx$, and $x = q$ by market feasibility, then

$$dS = [P(x) - C'(q)]dx$$

- *Intuition*:
  - The change in surplus of a marginal increase in consumption (and production) reflects the difference between the consumers' additional utility and firms' additional cost of production.

Welfare Analysis

- Differential change in surplus



Welfare Analysis

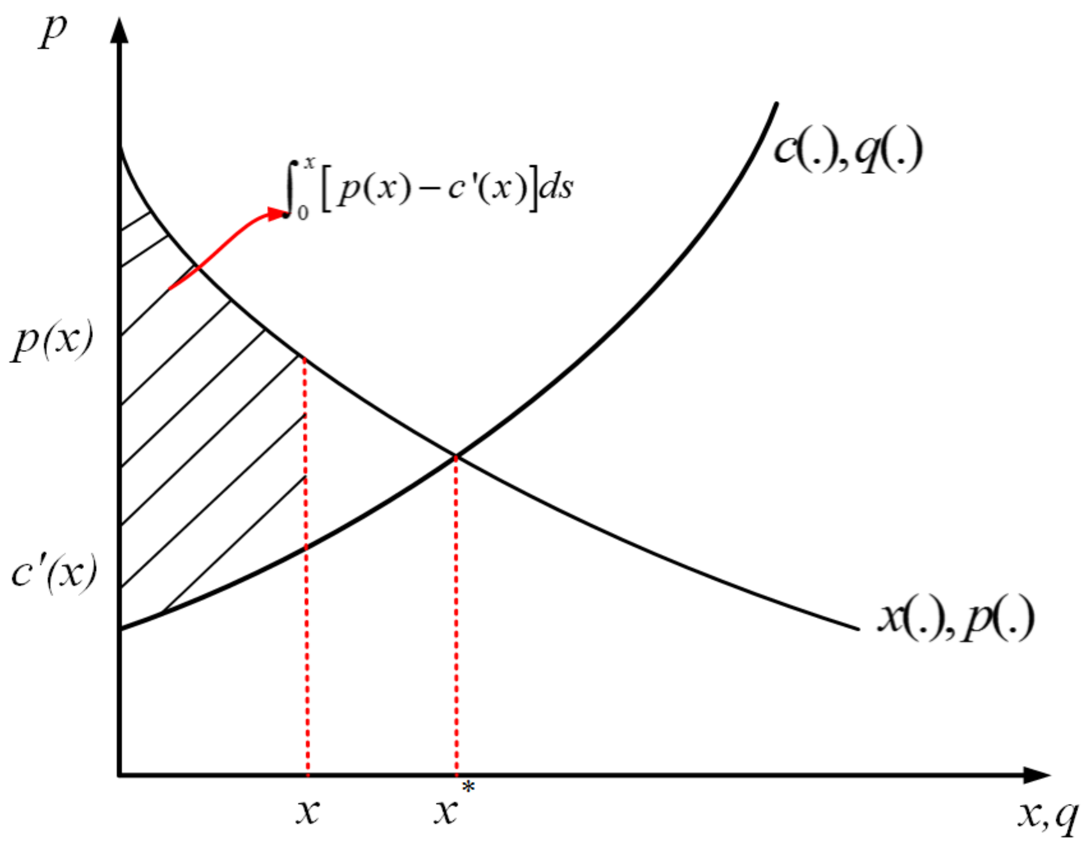
- We can also integrate the above expression, eliminating the differentials, in order to obtain the total surplus for an aggregate consumption level of x:

$$S(x) = S_0 + \int_0^x [P(s) - C'(s)]ds$$

where $S_0 = S(0)$ is the constant of integration, and represents the aggregate surplus when aggregate consumption is zero, $x = 0$.
  - $S_0 = 0$ if the intercept of the marginal cost function satisfies $c_j'(0) = 0$ for all J firms.

Welfare Analysis

- Surplus at aggregate consumption x



Welfare Analysis

- For which consumption level is aggregate surplus $S(x)$ maximized?
  - Differentiating $S(x)$ with respect to x,

$$S'(x) = P(x^*) - C'(x^*) \leq 0$$

$$\text{or,} \quad P(x^*) \leq C'(x^*)$$

  - The second order (sufficient) condition is

$$S''(x) = \underbrace{P'(x^*)}_{-} - \underbrace{C''(x^*)}_{+} < 0$$

    - Hence, $S(x^*)$ is concave.
    - Then, when $x^* > 0$, aggregate surplus $S(x)$ is maximized at $P(x^*) = C'(x^*)$.

Welfare Analysis

- Therefore, the CE allocation maximizes aggregate surplus.
- This is the ***First Welfare Theorem***:
  – Every CE is Pareto optimal (PO).

Welfare Analysis

- ***Example 6.4***:
  - Consider an aggregate demand $x(p) = a - bq$ and aggregate supply $y(p) = J \cdot \frac{p}{2}$, where J is the number of firms in the industry.
  - The CE price solves

$$a - bp = J \cdot \frac{p}{2} \quad \text{or} \quad p = \frac{2a}{2b+J}$$

  - Intuitively, as demand increases (number of firms) increases (decreases) the equilibrium price increases (decreases, respectively).

Welfare Analysis

- ***Example 6.4*** (continued):
  - Therefore, equilibrium output is

$$x^* = a - b\frac{2a}{2b+J} = \frac{aJ}{2b+J}$$

  - Surplus is

$$S(x^*) = \int_0^{x^*} p(x) - C'(x)dx$$

where $p(x) = \frac{a-x}{b}$ and $C'(x) = \frac{2x}{J}$.

  - Thus,

$$S(x^*) = \int_0^{x^*} \left(\frac{a-x}{b} - \frac{2x}{J}\right) dx = \frac{a^2 J}{4b^2 + 2bJ}$$

which is increasing in the number of firms J.